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Shroud Analysis

Texture Characterization by Means of the Wavelet Transform and New Ideas for Further Analysis

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Abstract

The aim of this work is to improve the digital images quality, in particular those relating to the Turin Shroud. Some comparisons are also proposed among different possible methods of image analysis.

The problem with Shroud images is that they are characterised by very low variations of the luminance of the various pixels. These almost unperceptible variations, cause great difficulty in analysing the images with the naked eye. Therefore some kind of elaboration becomes necessary.

The main effects causing noise are relative to the fabric, because it has spatial frequencies higher than the ones of the impressed image. It is clear that the value of spatial frequency becomes the most important parameter to make a distinction between the signal (meaning the image on the fabric) and the various defects.

The most important part of this work concerns the determination of the frequencies to be attributed to the vertical, horizontal and diagonal lines crossing the image. In fact, the work of cleaning two sample images proposed by [14], leaves some traces of the texture on the image. The basic idea is to apply the same algorithm prepared by [14] using the new frequencies found in this study, but avoiding soft or hard thresholding to eliminate the residual texture effects and avoiding the application of some color correction techniques.

The results of this study are presented in various chapters, starting from the two sample images which were used also in the work created by [14] (the one representing a spot and the one with the face of the man). Then the entire Shroud is analyzed, by means of a technique which is thoroughly explained in the dedicated chapters.

The initial part of this work can be regarded as an introduction to the problem and it presents some possibilities of future development of the study conducted so far, with some brief mentions to new theories or to other methods already developed in textile industry.

Introduction

The introduction is divided in several chapters and it takes into consideration various techniques of image analysis. Every chapter corresponds to a different method of analysis and highlights its main characteristics or hypothesis.

Some ideas of possible future studies concerning the Turin Shroud are proposed and discussed. Some of the methods treated have already been proved in textile industries.

In general, the best possible methods to solve some of the problems related to the Turin Shroud seem to concern:

- Textile industry, in terms of automated texture characteristics revelation;
- Space-varying and non-periodic signals, using a variety of mathematical methods of analysis.

1. Wavelet Analysis

This chapter is meant to focus on some properties of Wavelet Functions and some examples of famous wavelet functions are given. The examples presented in this section refer to various documents and information taken from the internet source. To see what web sites to visit to have the complete version of the theoretical aspects treated, it is possible to read the references in the specific section collecting the consulted web documentation.

1.1 Use of Complex and Real Wavelets

This paragraph is meant to focus on the principal differences and applications of the discrete wavelet transform, the complex wavelet transform and the continuous wavelet transform.

In general, the Discrete Wavelet Transform (DWT) produces watermark images with the best visual quality due to the absence of blocking artefacts. However, it has two drawbacks:

- The DWT lacks shift invariance, which means small shifts in input signal can cause big changes in the energy distribution of the wavelet coefficients.
- The DWT has poor directional selectivity for diagonal features.

There is only one filter for diagonal features. A straightforward way to provide shift invariance is to use the Undecimated Discrete Wavelet Transform (UDWT), but it is computationally expensive and still has poor selectivity for diagonal features, as it uses the same filters as the DWT.

The Complex Wavelet Transform, on the other hand, is more computationally efficient and has only a modest amount of redundancy; yet it provides approximate shift invariance and good directional selectivity.

Complex wavelets have not been widely employed in the past due to the difficulties in designing complex wavelets with perfect reconstruction (PR) properties.

The Continuous Wavelet Transform (CWT) is used to decompose a signal into wavelets, small oscillations that are highly localized in time. Whereas the Fourier transform decomposes a signal into infinite length sines and cosines, effectively losing all time-localization information, the CWT's basis functions are scaled and shifted versions of the time-localized mother wavelet. The CWT is used to construct a time-frequency representation of a signal that offers good time and frequency localization.

The CWT is an excellent tool for mapping the changing properties of non-stationary signals. The CWT is also an ideal tool for determining whether or not a signal is stationary in a global sense. When a signal is judged non-stationary, the CWT can be used to identify stationary sections of the data stream.

Let us consider as an example, the Morlet wavelet, defined as the product of a complex exponential wave and a Gaussian envelope:

$$\psi_0(\eta) = \pi^{-1/4} e^{i\omega_0 \eta} e^{-\eta^2/2}$$

Where Ψ_0 is the wavelet value at non-dimensional time η, and ω_0 is the wavenumber. This is the basic wavelet function, but we now need some way to change the overall size as well as slide the entire wavelet along in time. We thus define the "scaled wavelets" as:

$$\psi\left[\frac{(n'-n)\delta t}{s}\right] = \left(\frac{\delta t}{s}\right)^{1/2} \psi_0\left[\frac{(n'-n)\delta t}{s}\right]$$

where s is the "dilation" parameter used to change the scale, and n is the translation parameter used to slide in time. The factor of $s^{-1/2}$ is a normalization to keep the total energy of the scaled wavelet constant.

We are given a time series X, with values of x_n, at time index n. Each value is separated in time by a constant time interval dt. The wavelet transform $W_n(s)$ is just the inner product (or convolution) of the wavelet function with our original timeseries:

$$W_n(s) = \sum_{n'=0}^{N-1} x_{n'} \psi^*\left[\frac{(n'-n)\delta t}{s}\right]$$

where the asterisk (*) denotes complex conjugate.

The above integral can be evaluated for various values of the scale s (usually taken to be multiples of the lowest possible frequency), as well as all values of n between the start and end dates. A two-dimensional picture of the variability can then be constructed by plotting the wavelet amplitude and phase.

This convolution can be accomplished directly, or via the FFT-based fast convolution. Note that the CWT is a continuous function except for the discrete data series x and its discrete Fourier transform.

In the CWT, for each value of the scale used, the correlation between the scaled wavelet and the successive segments of the data stream is computed. Unless reconstruction is needed, there are no restrictions in the CWT as to how many scales are used, nor of the spacing between the scales. If needed, a high resolution spectrum can be generated for a narrow range of frequencies.

The convolutions can be done up to N times at each scale, and must be done all N times if the FFT is used. The CWT consists of N spectral values for each scale used, each of these requiring an inverse FFT. The computational load of the CWT and its memory requirements are thus considerable. The benefit from this high measure of redundancy in the CWT is an accurate time-frequency spectrum.

1.2 Wavelet Basis Functions (Mother Wavelets)

Unlike a Fourier decomposition which always uses complex exponential (sine and cosine) basis functions, a wavelet decomposition uses a time-localized oscillatory function as the analyzing or mother wavelet. The mother wavelet is a function continuous in both time and frequency and it serves as the source function from which scaled and translated basis functions are constructed. The mother wavelet can be complex or real, and it generally includes an adjustable parameter which controls the properties of the localized oscillation.

Wavelet analysis is more complicated than Fourier analysis since one must fully specify the mother wavelet from which the basic functions will be constructed. Below are some examples of Wavelet Mother Functions.

Morlet

The most commonly used CWT wavelet is the Morlet wavelet, a Gaussian-windowed complex sinusoid defined as following in the time and frequency domains:

$$\Psi_0(\eta) = \pi^{-1/4} e^{im\eta} e^{-\eta^2/2}$$

$$\hat{\Psi}_0(s\omega) = \pi^{-1/4} H(\omega) e^{-(s\omega - m)^2/2}$$

In these equations, ‘η’ is a non-dimensional time parameter, ‘m’ is the wavenumber, and ‘H’ is the Heaviside function.

In the time domain plot that follows, the Morlet wavelet is shown with an adjustable parameter m (wavenumber) of value six. This is the smallest wavenumber that allows an accurate signal reconstruction. The white curve is the real component and the cyan curve is the complex component. The Gaussian's second order exponential decay results in very good time localization:

The frequency domain representation is a single symmetric Gaussian peak. While not the sharp spectral peak of a sinusoid, the frequency localization is very good:

A Morlet wavelet with an adjustable parameter of twenty has a very different time domain representation:

Paul Wavelet

$$\Psi_0(\eta) = \frac{2^m i^m m!}{\sqrt{\pi(2m)!}}(1 - i\eta)^{-(m+1)}$$

$$\hat{\Psi}_0(s\omega) = \frac{2^m}{\sqrt{m(2m-1)!}} H(\omega)(s\omega)^m e^{-s\omega}$$

The Paul wavelet decays more quickly (as the square root of a factorial function) and enables much simpler wavelet structures that support reconstruction. At an order of four, only a single oscillation exists within the wavelet:

Even at the order of forty, the Paul wavelet evidences fewer oscillations than the Morlet at its lowest wavenumber:

The Paul wavelet offers better time localization than the Morlet because of its faster time-domain decay. Unlike the Morlet, however, its Fourier representation is not a symmetric peak but rather a peak that is right shifted toward higher frequencies. It approximately follows a gamma peak shape, and becomes less skewed as orders increase. Even at the Paul's order of 40, a significant right skew is evident in its frequency domain peak. The Paul wavelet thus offers poorer frequency resolution.

Derivative of Gaussian Wavelet

Another Mother wavelet is the Gaussian Derivative. The real component is defined as follows:

$$\Psi_0(\eta) = \frac{(-1)^{m+1}}{\sqrt{\Gamma\left(m+\frac{1}{2}\right)}} \frac{d^m}{d\eta^m}\left(e^{-\eta^2/2}\right)$$

$$\hat{\Psi}_0(s\omega) = -\frac{i^m}{\sqrt{\Gamma\left(m+\frac{1}{2}\right)}}(s\omega)^m e^{-(s\omega)^2/2}$$

The complex wavelet is generated by the addition of a Heaviside function in the frequency domain. This wavelet decays with the square root of the gamma function. Its time localization is between that of the Morlet and Paul wavelets. The lowest derivative, 2, is known as the Marr or Mexican Hat wavelet. A single oscillation is contained within the wavelet:

At its adjustable parameter of 80, the Gaussian Derivative wavelet contains about the same oscillation count as the Morlet with a wavenumber of twelve:

The Gaussian Derivative wavelet's frequency domain representation is exactly a gamma peak shape. The right shifted skew of the frequency peak is appreciably less than that of the Paul, and essentially vanishes above derivative order twenty or so (higher values for the derivative of the Gaussian Derivative wavelet produce nearly Gaussian frequency peaks).

1.3 Time Resolution versus Frequency Resolution

In the three cases discussed, the oscillations are effectively localized in time by rapid decays. Thus all three of the wavelets offer very good time localization.

The Paul wavelet localizes most efficiently in the time domain, the Gaussian Derivative wavelet is slightly less efficient, and the Morlet is the least efficient of the three (although a Gaussian decay still represents an excellent localization). The greatest differences tend to be in the frequency domain. There, for a given count

of evident oscillations in the wavelet, the Morlet offers the best frequency localization, the Gaussian Derivative wavelet is slightly less efficient, and the Paul wavelet will be least efficient.

This can be better illustrated graphically. The following contour plots use a data set of Gaussian noise where a single sinusoidal oscillation occurs between 0.12 and 0.16 seconds. The wavelet peak has been zoomed in to illustrate this time-frequency resolution tradeoff. The signal preceding the contour plots is a sinusoid with a white noise superimposed, with SNR= 20 dB. The plots refer to the same signal, but they consider just one oscillation, between 0.12 and 0.16 seconds.

Figure 1.3.1: Sample signal used for the time-frequency resolution discussion.

All plots use the same time-frequency scaling and a 12dB range z-gradient. All plots present time on x-axis and frequency on y-axis. The first three plots below are for the Morlet wavelet at wavenumbers of six, twelve, and twenty:

The next three are for the Paul at orders of four, sixteen, and forty:

The final three are for the Gaussian Derivative at orders two, twenty, and eighty:

In general, unless you need the best localization in time possible, the Morlet wavelet will probably suffice since it offers a very good resolution in both time and frequency. Its frequency resolution is respectable even at the lowest wavenumber.

The Paul wavelet's resolution in frequency at its best (order forty) is close to the Morlet with its least wavenumber of six. You can also see the asymmetry in frequency in each of the Paul wavelet examples. If one needs the best possible time localization, however, the Paul wavelet is a good choice.

The Gaussian Derivative wavelet at its best frequency resolution (order eighty) is close to the Morlet with a wavenumber of twelve. Unlike the Paul, higher orders of the Gaussian Derivative wavelet are essentially symmetric, and they can match the Morlet's frequency resolution with lower wavenumbers.

1.4 Multiresolution Analysis

In the Short Time Fourier Transform, a fixed width segment size controls the time-frequency resolution tradeoff. This results in a single resolution in time and a single resolution in frequency, regardless of the frequency being rendered. In contrast, wavelet analysis is a multiresolution method. The time-frequency resolution is not constant, but it varies with frequency. Multiresolution analysis is particularly useful where high frequency components exist for short durations within a signal, and low frequency components are more persistent.

Short-lived high frequency components need strong time localization. In order to achieve this, the frequency resolution of high frequency components will be diminished. On the other hand, long-lived low frequency components can tolerate poorer time resolution, but require effective frequency resolution. Low frequency components often determine the major part of a signal's character, and these properties will be best quantified if the frequency resolution is as fine as possible.

With multiresolution analysis, the difficult problem of determining an optimum segment width is avoided. The width of the time segment is automatically varied with frequency. The obvious drawback is that it is possible for a signal to contain short term low frequency components or long term high frequency components. For such signals, the Short Time Fourier Transform is probably a better choice than wavelet analysis. Further, this variation in frequency resolution makes it impossible to read spectral powers or amplitudes directly from a wavelet spectrum. This is another instance where the single resolution analysis of the Short Time Fourier Transform may be preferable.

Clearly, the peak height does not directly reflect the power or amplitude of the components. Despite the drawback of having to integrate the wavelet surface in order to find component powers, wavelet analysis does offer accurate power determination. Also, it is common practice to use wavelet analysis to identify stationary time segments within a signal. Once this is done, any of the frequency domain spectral methods (which require wide sense stationary data) can be used to process that segment. The Wavelet Filtering and Reconstruction can also be used to reconstruct the data associated with a stationary region of time-frequency space.

Focus on Complex versus Real Wavelets

A real wavelet consists of the real component of complex wavelet in the time domain. The frequency domain transform of a real wavelet is symmetric about frequency 0 and contains two peaks. The nature of an oscillation with the CWT spectrum will vary greatly with whether the wavelet is real or complex. The complex wavelet will evidence a constant power across the time duration of the oscillation. On the other hand, a real wavelet produces power only at those times where the oscillation is at an extreme or where a sharp discontinuity occurs.

The following example is for a frequency fifty sinusoid analyzed with an order four real Paul wavelet. Note the correspondence between peak extrema in the time domain and spectral power in the time-frequency domain:

At higher frequencies, the spectrum from a real wavelet lacks the resolution in time to map the peak extrema.

1.5 The CWT Spectrum

The CWT spectrum is computed by first taking a discrete Fourier transform of the data series. Then for each scale (specified frequency) in the spectrum, the daughter wavelet's frequency response is analytically computed and it is multiplied by the data's frequency transform and an inverse is taken of the product.

Fourier transforms assume periodic data. For the FFT fast convolution to be free of wraparound effects that arise as a consequence of non-periodicity in both the data and the response function (daughter wavelet), zero padding is needed equal to half the length of the non-zero elements in the daughter wavelet's frequency response. This length will vary as a function of scale (frequency).

Power is reduced near the edges of the spectrum with the introduction of the zeros into the convolution. This zone of edge effects is known as the cone of influence; thus spectral information within the cone of influence is not likely to be as accurate.

Unlike a Fourier spectrum where noise peaks directly reflect their power, the peaks in the wavelet power spectrum will vary as a consequence of the multiresolution property of the CWT. The wavelet spectral maxima will vary with the extent to which each peak is bidirectionally smeared in time-frequency space. There is also the issue of frequency sampling. Unlike the FFT, where a large data size automatically results in a high frequency density, the density of the CWT spectral frequencies is limited. The true spectral peak may fall between sampled frequencies, resulting in a lower detected maximum.

Orthogonal Wavelets and the DWT

Most wavelet analysis uses a pair of filters to successively isolate low and high pass components of a signal. This is known as the Discrete Wavelet Transform (DWT). The DWT wavelets are not continuous functions of time and their transforms are not a continuous function of frequency. Rather they are sets of time-domain filter coefficients that generally produce an orthogonal basis that greatly simplifies data filtering and reconstruction. A DWT is non-redundant. The number of blocks of wavelet power at each scale is a function of non-overlapping wavelet width. In a typical DWT, frequencies are spaced at unit powers of two and the count of blocks in time will increase by unit powers of two as these fixed frequencies increase. Although the DWT is fast and its time-frequency representation of a signal requires only modest memory, it is not practical for time-frequency spectral analysis.

1.6 Possible future development: Best Wavelet Determination

The problem in using Wavelets to analyze a signal is related to the correct choice of a suitable mother wavelet. In fact, if one wants to make the best possible analysis, the Wavelet function chosen has to resemble the spatial varying signal closely. Actually no general method has been proposed to optimize the Wavelet function to be used in each particular case.

One way to improve the efficacy of the algorithm proposed in this work is to use a genetic routine to determine the best mother wavelet to be used for each particular data sample.

The algorithm should be designed considering the typical signal and some 'genes' should be found to build a wavelet family for the particular case examined.

Finally, a specific performance function should be prepared, to help choosing the best Wavelet possible, selecting the 'genes' previously and appropriately determined.

Following this method, a more accurate choice would be done, instead of simply trying to find the Wavelet function which best fits the signal in a particular window. In fact this seems to be the greatest problem of the method of analysis proposed in this chapter: the necessity to determine the best wavelet shape and its parameters following an absolutely non-automated procedure.

Something has already been done on this topic, in fact Grace Chang, Martin Vetterli and Bin Yu have already published an article on the Journal of Transaction on Image Processing about adaptive wavelet thresholding for image denoise in the year 2000.

Therefore, this search for adaptive wavelet function shapes could really be the future of wavelet analysis.

2. The Huang-Hilbert Transform (HHT)

In the year 2000, a new analysis tool was patented through NASA. This new method, called the Hilbert-Huang Transform (HHT), combined the theories of both the Empirical Mode Decomposition and the Hilbert method.

It was found that this combination could be used to filter disturbances from non-stationary signal data and has been applied successfully in aeronautics, biology, physics, and many other fields. This method was beneficial because it not only can find the varying components of the data using the theories of Empirical Mode Decomposition (EMD), but could be used to create Hilbert spectrograms.

With collaborations with NASA, HHT software was developed as a "Computer Implemented Empirical Mode Decomposition."

2.1 Conceptual Introduction

Traditional data-analysis methods are all based on linear and stationary assumptions. Only in recent years have new methods been introduced to analyze nonstationary and nonlinear data. For example, wavelet analysis and the Wigner-Ville distribution were designed for linear but nonstationary data.

Additionally, various nonlinear time-series-analysis methods were designed for nonlinear but stationary and deterministic systems.

Unfortunately, in most real systems, the data are most likely to be both nonlinear and nonstationary. A necessary condition to represent nonlinear and nonstationary data is to have an adaptive basis. An a priori defined function cannot be relied on as a basis, no matter how sophisticated the basis function might be.

A few adaptive methods are available for signal analysis. For nonstationary and nonlinear data, where adaptation is absolutely necessary, no available methods can be found.

Being adaptive means that the definition of the basis has to be data-dependent, an a posteriori-defined basis. Even though challenging, new methods to examine data from the real world are certainly needed.

The HHT (Huang-Hilbert Transform) consists of two parts: empirical mode decomposition (EMD) and Hilbert spectral analysis (HSA). This method is potentially viable for nonlinear and nonstationary data analysis, especially for time-frequency-energy representations.

Powerful as it is, this method is entirely empirical. In order to make the method more robust and rigorous, many outstanding mathematical problems related to the HHT method need to be solved.

2.2 A brief Mathematical Description

The development of the HHT was motivated by the need to describe nonlinear distorted waves in detail, along with the variations of these signals that naturally occur in nonstationary processes.
To examine data from real-world nonlinear, nonstationary and stochastic processes, new approaches are urgently needed, for nonlinear processes need special treatment.
Other then periodicity, the detailed dynamics in the processes from the data need to be determined because one of the typical characteristics of nonlinear processes is their intra-wave frequency modulation, which indicates the instantaneous frequency changes within one oscillation cycle.

As Huang et al. (1998) pointed out, this intra-wave frequency variation is the hallmark of nonlinear systems.
In the past, when the analysis was based on the linear Fourier analysis, this intra-wave frequency variation could not be depicted, except by resorting to harmonics. Thus, any nonlinear distorted waveform has been referred to as “harmonic distortions.”
Harmonics distortions are a mathematical artifact resulting from imposing a linear structure on a nonlinear system. They may have mathematical meaning, but they certainly do not have a physical meaning (Huang et al. 1999).
The physically meaningful way to describe the system is in terms of the instantaneous frequency, which will reveal the intra-wave frequency modulations.
The easiest way to compute the instantaneous frequency is by using the Hilbert transform, through which the complex conjugate y(t) of any real valued function x(t) of Lp class can be determined by:

$$\mathcal{H}[x(t)] = \frac{1}{\pi}\,\mathrm{PV}\int_{-\infty}^{\infty}\frac{x(\tau)}{t-\tau}\,d\tau\,,$$

in which the PV indicates the principal value of the singular integral. With the Hilbert transform, the analytic signal is defined as

$$z(t) = x(t) + iy(t) = a(t)e^{i\theta(t)} ,$$

where

$$a(t) = \sqrt{x^2 + y^2}, \qquad \text{and} \qquad \theta(t) = \arctan\left(\frac{y}{x}\right) .$$

Here, a(t) is the instantaneous amplitude, and θ is the phase function, and the instantaneous frequency is simply:

$$\omega = \frac{d\theta}{dt} .$$

The Hilbert transform is the convolution of x(t) with 1/t. The polar coordinate expression further clarifies the local nature of this representation: it is the best local fit of an amplitude and phasevarying trigonometric function to x(t).

2.3 The empirical mode decomposition method (the sifting process)

As discussed by Huang et al. (1996, 1998, 1999), the empirical mode decomposition method is necessary to deal with data from nonstationary and nonlinear processes.

In contrast to almost all of the previous methods, this new method is intuitive, direct and adaptive, with an a posteriori-defined basis, from the decomposition method, based on and derived from the data. The decomposition is based on the simple assumption that any data consists of different simple intrinsic modes of oscillations.

Each intrinsic mode, linear or nonlinear, represents a simple oscillation, which will have the same number of extrema and zero-crossings. Furthermore, the oscillation will also be symmetric with respect to the "local mean." At any given time, the data may have many different coexisting modes of oscillation, one

superimposing on the others. The result is the final complicated data. Each of these oscillatory modes is represented by an intrinsic mode function (IMF) with the following definition:

1) in the whole dataset, the number of extrema and the number of zero-crossings must either equal or differ at most by one;

2) at any point, the mean value of the envelope defined by the local maxima and the envelope defined by the local minima is zero.

The IMF can have a variable amplitude and frequency as functions of time.

With the above definition for the IMF, one can then decompose any function as follows:

- Take the test data;
- Identify all the local extrema;
- Connect all the local maxima by a cubic spline, as shown in the upper envelope;
- Repeat the procedure for the local minima to produce the lower envelope.

The upper and lower envelopes should cover all the data between them. Their mean is designated as m1 and the difference between the data and m1 is the first component h1:

$$h1 = x(t) - m1$$

Ideally, h1 should satisfy the definition of an IMF, for the construction of h1 described above should have made it symmetric and have all maxima positive and all minima negative. However, even if the fitting is perfect, a gentle hump on a slope can be amplified to become a local extremum in changing the local zero from a rectangular to a curvilinear coordinate system.

After the first round of sifting, the hump may become a local maximum. New extrema generated this way actually reveal the proper modes lost in the initial examination.

In fact, with repeated siftings, the sifting process can recover signals representing low-amplitude riding waves.

The sifting process serves two purposes: to eliminate riding waves and to make the wave profiles more symmetric.

While the first purpose must be achieved for the Hilbert transform to give a meaningful instantaneous frequency, the second purpose must also be achieved in case the neighboring wave amplitudes have too large a disparity.

Toward these ends, the sifting process has to be repeated as many times as required to reduce the extracted

signal to an IMF. In the subsequent sifting processes, h1 can be treated only as a proto-IMF. In the next step, it is treated as the data; then,

$$h11 = h1 - m11.$$

After repeated siftings in this manner, up to k times, h1k becomes an IMF, that is:

$$h1k = h1(k-1) - m1k\ ;$$

then, it is designated as c1 = h1k . Here, a critical decision must be made: the stoppage criterion.

Historically, two different criteria have been used: the first one was used in Huang et al. (1998). This stoppage criterion is determined by using a Cauchy type of convergence test. Specifically, the test requires the normalized squared difference between two successive sifting operations defined to be small.

$$SD_k = \frac{\sum_{t=0}^{T} |h_{k-1}(t) - h_k(t)|^2}{\sum_{t=0}^{T} h_{k-1}^2}$$

If this squared difference SD_k is smaller than a predetermined value, the sifting process will be stopped.

This definition seems to be rigorous, but it is very difficult to implement in practice. Two critical questions need to be solved: first, the question of how small is small enough. Second, this criterion does not depend on the definition of the IMFs. The squared difference might be small, but nothing guarantees that the function will have the same numbers of zero-crossings and extrema, for example.

These shortcomings prompted Huang et al. (1999, 2003) to propose a second criterion based on the agreement of the number of zero-crossings and extrema.

Specifically, a S-number is pre-selected. The sifting process will stop only after S consecutive times, when the numbers of zero-crossings and extrema stay the same and are equal or differ at most by one. This second choice has its own difficulty: how to select the S number. Obviously, any selection is ad hoc, and a rigorous justification is needed.

In a recent study of this open-ended sifting, Huang et al. (2003) used the many possible choices of S numbers to form an ensemble of IMF sets, from which an ensemble mean and confidence were derived. Furthermore, through comparisons of the individual sets with the mean, Huang et al. established an empirical guide. For the optimal siftings, the range of S-numbers should be set between four and eight.

Now assume that a stoppage criterion was selected, and that the first IMF c1 was found. Overall, c1 should contain the finest scale or the shortest period component of the signal. It follows that c1 can be separated from the rest of the data by

$$r1 = x(t) - c1.$$

Since the residue r1 still contains longer period variations in the data, it is treated as the new data and subjected to the same sifting process as described above. This procedure can be repeated with all the subsequent rj's, and the result is

$$r2 = r1 - c1.$$

The sifting process can be stopped finally by any of the following predetermined criteria:

1) either when the component c_n or the residue r_n becomes so small that it is less than the predetermined value of substantial consequence;
2) or when the residue r_n becomes a monotonic function from which no more IMFs can be extracted.

Even for data with zero mean, the final residue still can be different from zero. If the data have a trend, the final residue should be that trend. By summing up we finally obtain:

$$x(t) = \sum_{j=1}^{n} c_j + r_n.$$

Thus, a decomposition of the data into n-empirical modes is achieved, and a residue r_n obtained which can be either the mean trend or a constant.

As discussed here, the EMD technique needs only the locations of the local extrema.

The components of the EMD are usually physically meaningful, for the characteristic scales are defined by the physical data.

2.4 The Hilbert Spectral Analysis

Having obtained the intrinsic mode function components, one will have no difficulty in applying the Hilbert transform to each IMF component, and in computing the instantaneous frequency. After performing the Hilbert transform on each IMF component, the original data can be expressed as the real part R in the following form:

$$x(t) = \Re\left\{\sum_{j=1}^{n} a_j(t)\exp\left[i\int \omega_j(t)\,dt\right]\right\}$$

Here, the residue r_n has been left out on purpose, for it is either a monotonic function or a constant.

In consideration of the uncertainty of the longer trend, and in the interest of obtaining the information contained in the other low-energy but clearly oscillatory components, the final non-IMF component should be left out.

However, it could be included if physical considerations justify its inclusion.

The equation above gives both the amplitude and frequency of each component as a function of time. The same data expanded in a Fourier representation would be:

$$x(t) = \Re\left[\sum_{j=1}^{n} a_j e^{i\omega_j(t)t}\right],$$

with both a_j and ω_j as constants.

The contrast between these two formulae is clear: the IMF represents a generalized Fourier expansion. The variable amplitude and the instantaneous frequency have not only greatly improved the efficiency of the expansion, but also enabled the expansion to accommodate nonlinear and nonstationary data.

With the IMF expansion, the amplitude and frequency modulations are also clearly separated. Thus, the restriction of the constant amplitude and fixed frequency of the Fourier expansion has been overcome, with a variable amplitude and frequency representation.

This frequency-time distribution of the amplitude is designated as the “Hilbert amplitude spectrum” H(ω, t), or simply “Hilbert spectrum.”

With the Hilbert Spectrum defined, we can also define the marginal spectrum h(ω) as:

$$h(\omega) = \int_0^T H(\omega, t)\, dt$$

The marginal spectrum offers a measure of the total amplitude (or energy) contribution from each frequency value. This spectrum represents the accumulated amplitude over the entire data span in a probabilistic sense.

The combination of the empirical mode decomposition and the Hilbert spectral analysis is also known as the "Hilbert–Huang transform" (HHT).

In this case there is no uncertainty principle limitation on time or frequency resolution from the convolution pairs based also on a priori basis.

A comparative summary of Fourier, wavelet and HHT analyses is given in the following table:

	Fourier	Wavelet	Hilbert
Basis	A priori	A priori	Adaptive
Frequency	Convolution: global uncertainty	Convolution: regional uncertainty	Differentiation: local, certainty
Presentation	Energy - frequency	Energy–time-frequency	Energy–time-frequency
Nonlinear	no	no	yes
Nonstationary	no	yes	yes
Feature extraction	no	Discrete: no, continuous: yes	yes
Theoretical base	Theory complete	Theory complete	Empirical

It is possible to see that HHT is based on an adaptive basis. The frequency is derived by differentiation rather than convolution; therefore, it is not limited by the uncertainty principle; it is applicable to nonlinear and nonstationary data and presents the results in time-frequency-energy space for feature extraction.

2.5 Analytical Examination of the EMD Method

To understand why the spline fitting of local extrema is able to extract the simplest components, let us consider two main cases of signal composition: the sum of an harmonic wave plus the DC offset (1) and the sum of two different harmonics (2).

In the case of the DC offset, the spline fitting of maxima $\omega t = 0 \pm 2\pi n$ produces a straight line:

$$A_{max} = A1(t)+a(t)$$

and interpolation between minima $\omega t = \pi \pm 2\pi n$ produces another straight line:

$$A_{min} = -A1(t)+a(t).$$

The mean value of these two lines will generate back the initial offset function $0.5(A_{max}+A_{min}) = a(t)$.

Extracting the obtained offset from the initial composition yields the initial harmonic:

$$x(t) - a(t) = A1(t) \cos(\omega t).$$

In this manner the EMD method upon each iteration successfully separates the harmonics and the slow aperiodic components.

Now let us consider the frequency properties of the extreme lines. In general, the zero crossing frequency estimated locally can be considered as a low-pass filtered global IF.

Let a harmonic wave with the frequency f0 (Hz) be sampled with the sample frequency Fs. During a period the number of samples describing the main and the higher harmonics is equal to Fs/f0. In theory, samples can depict frequencies higher than the main harmonic frequency f0 up to the Nyquist frequency 0.5Fs.

Naturally the global IF along the period also consists of the same number of samples. But the local zero crossing estimation during the single period will include two points only and cannot depict frequencies higher that the main harmonic frequency f0. In other words, the zero crossing estimation returns only the main harmonic frequency and reduces all higher frequencies. This means that the zero crossing function resembles a low-pass filtered IF function.

The relative cutoff frequency of the low-pass filter is equal to the relation between the main and the sample frequencies 2f0/Fs. Such a low-pass filtered IF will determine the frequency of the largest component. This means that the EMD method successfully decomposes both the amplitude and frequency of the largest component (the intrinsic function) from wideband signals.

Fig. 3. The bi-harmonic narrowband (a) and wideband (b) composition: the signal (—), the envelope (▬), the opposite sign envelope (··), the upper extreme line (- -), the lower extreme line (- · -).

Figure 2.1: MF Decomposition.

3. The Hough Transform

The Hough Transform has already been used for the automatic inspection of defects in directionally textured surfaces which arise in textile fabrics and machined surfaces.

The method does not rely on local features of textures; it is based on a global image restoration scheme using the Fourier transform. The line patterns of any directional textures in the spatial domain image are removed by detecting the high-energy frequency components in the Fourier domain image using a one-dimensional Hough transform, setting them to zero, and finally back-transforming to a spatial domain image.

A statistical process control scheme is used to set up the control limits for discriminating between defects and homogeneous line patterns. The experiments on a variety of textile fabrics, machined surfaces and natural wood have shown the effectiveness of this kind of method.

Textures are generally classified into two major types: structural and statistical.

Structural textures are composed of repetitions of some basic texture primitives with a deterministic rule of displacement, whereas statistical textures cannot be described by primitives and displacement rules. The spatial distribution of gray levels in such textured image is rather stochastic. A directional texture is a homogeneous texture that consists of an arrangement primarily of line structures appearing periodically on the surface.

Here after are some examples of the results obtained in some applications and studies.

Figure 3.1: A directional texture with a single line-direction. (a)original image;(b)Fourier domain image displayed as an intensity function; (c)the A(u) histogram in the Hough transform space; (d)restored image from the IFT.

Figure 3.2: A directional texture with three line-directions. (a)original image;(b)Fourier domain image; (c)the A(u) histogram in the Hough transform space; (d)restored image from the IFT.

The Fourier spectrum is ideally suited for describing the directionality of periodic (or almost periodic) line patterns in gray-level images.

The directional characters of a gray-level image clearly correspond to high-energy frequency components, which are distributed along the straight lines in the Fourier domain image with directions orthogonal to their counterparts in the spatial domain image.

The lines associated with high-energy frequency components in the Fourier domain image are detected by using the Hough transform (HT).

The frequency components falling in the neighborhood of the detected lines in the Fourier domain image are virtually eliminated by setting them to zero. After clearing the specific regions, we back-transform the Fourier domain image using the inverse Fourier transform (IFT).

The IFT process will remove all homogeneous, directional textures in the original gray-level image.

3.1 Removal of line patterns : The Fourier transform

The Fourier-based image restoration technique is used to remove the line patterns in textured surfaces.

The FT has the desirable properties of noise-immunity, orientation preserving, and enhancement of periodic features. The FT characterizes the surface image in terms of frequency components.

The periodically occurring features, such as structural lines, can be observed from the magnitude of frequency components.

These global texture patterns are easily distinguishable as concentration of high-energy bursts in the spectrum. The FT is rotation-dependent, i.e. rotating the original image by an angle will rotate its corresponding frequency plane by the same angle. Further, the transform of a horizontal line in the spatial domain image appears like a vertical line in the Fourier domain image. A line in the spatial domain image and its transformed counterpart in the Fourier domain image are orthogonal to each other.

The line pattern information in the spatial domain image is highly concentrated on the high-energy frequency components distributed along the transformed lines in the Fourier domain image.

To remove the line patterns, we can edit the Fourier domain image and set those high-energy frequency components to zero; then apply the IFT to obtain a uniform gray-level image. Following the idea presented here, the slope angles of the transformed lines associated with high-energy frequency components in the Fourier domain image are automatically detected using the HT technique.

3.2 What does the Hough Transform perform?

The Hough transform is a feature extraction technique used in image analysis, computer vision, and digital image processing. The purpose of the technique is to find imperfect instances of objects within a certain class of shapes by a voting procedure.

This voting procedure is carried out in a parameter space, from which object candidates are obtained as local maxima in a so-called accumulator space that is explicitly constructed by the algorithm for computing the Hough transform.

The classical Hough transform was concerned with the identification of lines in the image, but later the Hough transform has been extended to identify positions of arbitrary shapes. The transform was popularized in the computer vision community by Dana H. Ballard through a 1981 journal article titled "Generalizing the Hough transform to detect arbitrary shapes".

In HT, a voting process transforms the set of points in the image space into a set of accumulated votes in the parameter space. Therefore, the HT converts a difficult global shape detection problem in the image space into an easily solved local peak detection problem in the parameter space.

As mentioned previously, the Fourier spectrum is symmetric about the central component and every transformed line in the Fourier domain image will pass through the central frequency point ($u0$; $v0$).

Therefore, one parameter (slope angle u) is sufficient to specify a straight line that passes through the center of the spectrum, by shifting the origin to the center of the image.

The parameter space of the HT is simply one-dimensional. The parameter u is quantized to 180 levels. The quantized u region is treated as a 1D array of accumulator A(u), initially set to zero.

The HT is to scan all pixels in the Fourier domain image. At every high-energy pixel (u,v), we compute the slope angle:

$$\theta = \begin{cases} \tan^{-1}\left(\dfrac{v - v_0}{u - u_0}\right) & \text{if} \dfrac{v - v_0}{u - u_0} > 0 \\ 180 + \tan^{-1}\left(\dfrac{v - v_0}{u - u_0}\right) & \text{if} \dfrac{v - v_0}{u - u_0} < 0 \end{cases}$$

and increment the corresponding accumulator A(u) by one.

Thus, a given u in the 1D accumulator array eventually records the total number of points which lie on the line v = atan(θ)*u. A large peak in the accumulator corresponds to a significant line in the Fourier domain image.

Note that in the proposed HT the parameter u is directly computed from each selected pixel (u,v), and the accumulator array is one-dimensional.

Therefore, the line detection procedure is computationally efficient and requires only a small storage for the accumulator.

The image is transformed from top to bottom, i.e. (0,0) is on the upper-left of the image. Only the pixel with power spectrum P(u,v) larger than 120 is considered as a high-energy component, and is used for the computation of the slope angle u . The energy threshold 120 is empirically determined, as all high-energy frequency components associated with a line pattern have the magnitude over 120. The value of P(u,v) is scaled to the interval [0,225] for 8-bit gray-level display.

This threshold value can be loosely determined without affecting the detection result due to the nature of the HT.

For a noisy or less-periodically distributed texture, we may select a lower energy threshold so that every frequency point in the vicinity of the lines in the Fourier domain image can contribute one vote to the Hough

accumulator. In addition, the increment of a vote at (u,v) to the accumulator can be given by the power magnitude $P(u,v)$ instead of one. This will further distinguish the peak from random noise.

High-energy frequency components associated with periodical line patterns may appear around the detected principal lines in the Fourier domain image.

To completely eliminate all homogeneous line patterns in the spatial domain image, not only the frequency components lying on the principal lines defined by up*i*, but also those frequency components in the neighborhood of the principal lines must be removed from the Fourier domain image.

Conclusions

In automated surface inspection, one often has to solve the problem of detecting small defects which appear as local anomalies embedded in a homogeneous texture.

As local defects exhibit no distinct textural properties, this task is clearly different from segmentation by texture. Traditional local approaches for automated surface inspection compute a set of textural features in a sliding window and search for significant local deviations in the feature values between the sensed image and the model image.

The proposed method does not depend on local features of textures. It is based on a global image restoration scheme using the FT.

The line pattern of any directional texture in the spatial domain image can be easily removed by detecting the high-energy frequency components in the Fourier domain image, setting them to zero, and finally back-transforming to a spatial domain image. The transform of periodic lines of a specific orientation u in the spatial domain image appears a line orthogonal to u in the Fourier domain image. The line associated with high-energy frequency components in the Fourier domain image is detected using the HT.

In some experiments a variety of manmade surfaces with regular line patterns have already been examined, such as machined surfaces and textile fabrics, and even natural surfaces with less regular line patterns such as wood.

Comparison of the proposed method with feature-extraction and Gabor-filtering methods

Method	The proposed method	Feature-extraction method	Gabor-filtering method
Training	Very efficient (only require the number of directions in a texture sample)	Time-consuming and require human knowledge	Time-consuming (need to determine the optimal Gabor parameters)
Inspection	Efficient (one Fourier transform and one inverse Fourier transform)	Depend on the textural features and classifiers selected	Time-consuming (the convolution in a sliding window is computationally intensive)
Storage for model samples	One parameter N_θ (the number of directions)	The statistics of all feature values	The elements of the Gabor filter
Effect on rotation	Invariant	Depend on the features selected	Sensitive to rotation
Applicable textures	Directional textures only	Both structured and statistical textures	Both structured and statistical textures

The table above summarizes the qualitative comparison of the discussed method with feature-extraction methods and Gabor-filtering methods in terms of training and inspection complexity, storage requirement of model samples, effect on rotation, and the applicable texture types.

This method cannot be directly extended to the application of statistical textures such as cast surfaces as the frequency components associated with the homogeneous regions and defective regions are highly mixed together in the Fourier domain image.

It is difficult to manipulate the frequency components associated with homogeneous regions without affecting those components associated with defective regions.

3.3 A closer look to theory

The simplest case of Hough transform is the linear transform for detecting straight lines. In the image space, the straight line can be described as y = mx + b and can be graphically plotted for each pair of image points (x,y). In the Hough transform, a main idea is to consider the characteristics of the straight line not as image points x or y, but in terms of its parameters, here the slope parameter m and the intercept parameter b. Based on that fact, the straight line y = mx + b can be represented as a point (b, m) in the parameter space. However, one faces the problem that vertical lines give rise to unbounded values of the parameters m and b. For computational reasons, it is therefore better to parameterize the lines in the Hough transform with two other parameters, commonly referred to as r and θ (*theta*). The parameter r represents the distance between the line and the origin, while θ is the angle of the vector from the origin to this closest point. Using this parametrization, the equation of the line can be written as:

$$r = x\cos\theta + y\sin\theta.$$

It is therefore possible to associate to each line of the image, a couple (r,θ) which is unique. The (r,θ) plane is sometimes referred to as *Hough space* for the set of straight lines in two dimensions. An infinite number of lines can pass through a single point of the plane.

If the curves corresponding to two points are superimposed, the location (in the *Hough space*) where they cross correspond to lines (in the original image space) that pass through both points. More generally, a set of points that form a straight line will produce sinusoids which cross at the parameters for that line. Thus, the problem of detecting collinear points can be converted to the problem of finding concurrent curves.

3.4 Something about numerical implementation

As already said, the Hough transform algorithm uses an array, called accumulator, to detect the existence of a line y = mx + b. The dimension of the accumulator is equal to the number of unknown parameters of the Hough transform problem. For example, the linear Hough transform problem has two unknown parameters: m and b. The two dimensions of the accumulator array would correspond to quantized values for m and b. For each pixel and its neighborhood, the Hough transform algorithm determines if there is enough evidence of an edge at that pixel. If so, it will calculate the parameters of that line, and then look for the accumulator's bin that the parameters fall into, and increase the value of that bin. By finding the bins with the highest values, typically by looking for local maxima in the accumulator space, the most likely lines can be extracted, and their geometric definitions read off. The simplest way of finding these *peaks* is by applying some form of threshold, but different techniques may yield better results in different circumstances - determining which lines are found as well as how many. Since the lines returned do not contain any length information, it is often next necessary to find which parts of the image match up with which lines.

Moreover, due to imperfection errors in the edge detection step, there will usually be errors in the accumulator space, which may make it non-trivial to find the appropriate peaks, and thus the appropriate lines.

4. Fast Fourier Transform and Spectral Analysis

The Fourier Transform is an important image processing tool which is used to decompose an image into its sine and cosine components. The output of the transformation represents the image in the *Fourier* or frequency domain, while the input image is the spatial domain equivalent. In the Fourier domain image, each point represents a particular frequency contained in the spatial domain image.

The Fourier Transform is used in a wide range of applications, such as image analysis, image filtering, image reconstruction and image compression.

As we are only concerned with digital images, we will restrict this discussion to the *Discrete Fourier Transform* (DFT).

The DFT is the sampled Fourier Transform and therefore it does not contain all frequencies forming an image, but only a set of samples which is large enough to fully describe the spatial domain image. The number of frequencies corresponds to the number of pixels in the spatial domain image, *i.e.* the image in the spatial and Fourier domain are of the same size.

For a square image of size N×N, the two-dimensional DFT is given by:

$$F(k,l) = \frac{1}{N^2} \sum_{a=0}^{N-1} \sum_{b=0}^{N-1} f(a,b)\, e^{-\iota 2\pi (\frac{ka}{N} + \frac{lb}{N})}$$

where *f(a,b)* is the image in the spatial domain and the exponential term is the basis function corresponding to each point *F(k,l)* in the Fourier space. The equation can be interpreted as: the value of each point *F(k,l)* is obtained by multiplying the spatial image with the corresponding base function and summing the result.

The basic functions are sine and cosine waves with increasing frequencies, *i.e. F(0,0)* represents the DC-component of the image which corresponds to the average brightness and *F(N-1,N-1)* represents the highest frequency.

In a similar way, the Fourier image can be re-transformed to the spatial domain. The inverse Fourier transform is given by:

$$f(a,b) = \frac{1}{N^2} \sum_{k=0}^{N-1} \sum_{l=0}^{N-1} F(k,l)\, e^{\iota 2\pi (\frac{ka}{N} + \frac{lb}{N})}$$

To obtain the result for the above equations, a double sum has to be calculated for each image point. However, because the Fourier Transform is *separable*, it can be written as:

$$F(k,l) = \frac{1}{N} \sum_{b=0}^{N-1} P(k,b)\, e^{-\iota 2\pi \frac{lb}{N}}$$

where:

$$P(k,b) = \frac{1}{N}\sum_{a=0}^{N-1} f(a,b)\ e^{-\iota 2\pi \frac{ka}{N}}$$

Using these two formulas, the spatial domain image is first transformed into an intermediate image using *N* one-dimensional Fourier Transforms. This intermediate image is then transformed into the final image, again using *N* one-dimensional Fourier Transforms. Expressing the two-dimensional Fourier Transform in terms of a series of *2N* one-dimensional transforms decreases the number of required computations.

Even with these computational savings, the ordinary one-dimensional DFT has N^2 complexity. This can be reduced to $N\log_2 N$ if we employ the *Fast Fourier Transform* (FFT) to compute the one-dimensional DFTs. This is a significant improvement, in particular for large images. There are various forms of the FFT and most of them restrict the size of the input image that may be transformed, often to $N=2^n$ where *n* is an integer. The mathematical details are well described in the literature.

The Fourier Transform produces a complex number valued output image which can be displayed with two images, either with the *real* and *imaginary* part or with *magnitude* and *phase*. In image processing, often only the magnitude of the Fourier Transform is displayed, as it contains most of the information of the geometric structure of the spatial domain image. However, if we want to re-transform the Fourier image into the correct spatial domain after some processing in the frequency domain, we must make sure to preserve both magnitude and phase of the Fourier image.

The Fourier domain image has a much greater range than the image in the spatial domain. Hence, to be sufficiently accurate, its values are usually calculated and stored in float values.

To clarify some of the concepts treated and to have a better idea of how the two-dimensional FT applied to an image works, it is possible to have a look at the following examples.

The FT tries to represent all images as a summation of cosine-like images. Therefore images that are pure cosines have particularly simple FTs.

The picture above shows two images with their Fourier Transforms directly underneath. The images are a pure horizontal cosine of 8 cycles and a pure vertical cosine of 32 cycles.

Notice that the FT for each just has a single component, represented by 2 bright spots symmetrically placed about the center of the FT image. The center of the image is the origin of the frequency coordinate system. The u-axis runs left to right through the center and represents the horizontal component of frequency. The v-axis runs bottom to top through the center and represents the vertical component of frequency. In both cases there is a dot at the center that represents the (0,0) frequency term or average value of the image.

Images usually have a large average value (like 128) and lots of low frequency information so FT images usually have a bright blob of components near the center. Notice that high frequencies in the vertical direction will cause bright dots away from the center in the vertical direction. And that high frequencies in the horizontal direction will cause bright dots away from the center in the horizontal direction.

Here are two images of more general Fourier components. They are images of 2D cosines with both horizontal and vertical components. The one on the left has 4 cycles horizontally and 16 cycles vertically. The one on the right has 32 cycles horizontally and 2 cycles vertically. (Note: The gray band corresponds at the areas where the function goes through gray = 128 which happens twice/cycle.)

One may begin to notice the symmetry. For all real (as opposed to imaginary or complex) images, the FT is symmetrical about the origin so the 1st and 3rd quadrants are the same and the 2nd and 4th quadrants are the same. If the image is symmetrical about the x-axis (as the cosine images are) 4-fold symmetry results.

4.1 Magnitude and Phase

Note that f(x,y) is the image and it is real, but F(u,v) (abbreviate as F) is the FT and is, in general, complex. Generally, F is represented by its magnitude and phase rather that its real and imaginary parts, where:

MAGNITUDE(F) = SQRT(REAL(F)^2+IMAGINARY(F)^2)

PHASE(F) = ATAN(IMAGINARY(F)/REAL(F)).

Briefly, the magnitude tells "how much" of a certain frequency component is present and the phase tells "where" the frequency component is in the image. To illustrate this consider the following:

Note that the FT images we look at are just the magnitude images. The images displayed are horizontal cosines of 8 cycles, differing only by the fact that one is shifted laterally from the other by 1/2 cycle (or by PI in phase). Note that both have the same FT magnitude image. The phase images would be different, of course. It is wise to remember that when one looks at a common FT image and thinks about "high" frequency power and "low" frequency power, this is only the magnitude part of the FT.

4.2 Rotation and Edge Effects

In general, rotation of the image results in equivalent rotation of its FT. To see that this is true, we will take the FT of a simple cosine and also the FT of a rotated version of the same function. The result can be seen by observing the following picture:

The horizontal cosine has its normal, very simple FT. But the rotated cosine seems to have an FT that is much more complicated, with strong diagonal components, and also strong "plus sign" shaped horizontal and vertical components. The question is, where did these horizontal and vertical components come from? The answer is that the FT always treats an image as if it were part of a periodically replicated array of identical images extending horizontally and vertically to infinity. And there are strong edge effects between the neighbors of such a periodic array as can be seen by:

Thus, what we see as the FT in the "slant" image (lower right of the image before last) is actually the combination of the actual FT of the cosine function and that caused by the edge effects of looking at a finite part of the image. These edge effects can be significantly reduced by "windowing" the image with a function that slowly tapers off to a medium gray at the edge. The result can be seen by:

The windowed image is shown in the upper left. Its FT is shown in the lower left. The non-windowed FT is shown in the upper right and the actual, true FT of a cosine is shown in the lower right. These images are all scaled differently and the comparison is only qualitative, but it can be seen that the windowed image FT is much closer to the true FT and eliminates many of the edge effects.

4.3 Spectral Connotation of the body image

In this section, an application of the two-dimensional FFT to image analysis is illustrated.

Therefore, the method of analysis and the results presented are related to a spectral analysis made using the two-dimensional Fast Fourier Transform. This study is applied to some sample images of the Shroud, but the final aim is to demonstrate that there exist some specific frequencies representing the image and some others connected with the texture (background). The images chosen are the following ones:

Figure 4.3.1: Images chosen for two-dimensional FFT analysis.

Figure 4.3.2: Image illustrating the position of the two sample images on the Shroud.

The two samples presented have been chosen for the application of this preliminary study. Both the details are in gray levels and the tones conversion ratio is not known.

The picture on the left of the image 4.3.1 represents a spot and it has been chosen because the effect of the vertical and horizontal lines of the texture is clearly visible. This image is composed of 591 pixels in the horizontal direction and 537 pixels in the vertical direction, with a 25 pixel/cm resolution.

The picture on the right of the image 4.3.1 represents a human face, instead. It has been used in this study because it is considered to be one of the most interesting details to be analyzed, for which considerable improvement is desirable. This portion of the entire fabric is characterized by 661 pixels in the horizontal direction and 737 in the vertical direction, with a 25 pixel/cm resolution, as in the previous case.

In general, the body image impressed on the fabric is characterised by spatial frequencies lower than those relative to defects. This lack of high frequency components is due to the absence of some clearly visible objects or particulars in the image.

This means that the corresponding signal never resembles a step, in fact a step has some high frequency components in its FFT spectrum, but in this case they are not present. The consequence is that the pixels representing the body image in the FFT spectrum are the ones very next to the origin.

The reader is remanded to chapter 5 to have a deeper description of the Shroud Image characteristics, however the information written here is sufficient to understand the study related to FFT application.

To try and separate the image from the fabric, it is possible to eliminate the luminance of several pixels next to the origin, so as to obtain an image completely free from the image impressed. The software used to this extent is Matlab, which does not have high FFT visualization qualities, therefore the results obtained are purely qualitative. It is possible to create some more quantitative results by using a dedicated software, like for example *Optimas*.

The final step is the execution of the inverse FFT, obtaining the fabric without the some of the body image.

4.4 Results of the Separation of the Image from the Background using FFT

The results illustrated below are relative to a preliminar Fourier analysis of the two sample Shroud images shown in figure 4.3.1. In fact the method proposed is meant to separate the image from the background, by searching the specific frequencies corresponding to the image.

The method procedees by successive trials, eliminating from the Fourier spectrum obtained with Matlab function 'fft2', the frequencies which are thought to be representative of the image. Then, with function 'ifft2', it is possible to go back to the spatial domain, having left only the texture frequencies. After this, it is possible to subtract the texture image to the original image representing both the image impressed and the background, to have as a result, the image impressed without the texture.

As already specified, this has been done by means of Matlab, which is not a specific software for FFT or spectral analysis, therefore the results obtained and shown below are purely qualitative. This means that the frequencies used to separate the image frequencies from the texture background have been chosen after some trials, by eliminating the frequencies corresponding to certain pixels and by comparing the results obtained for the various trials to find an acceptable result.

The picture below offers a qualitative idea of the area in which the frequencies have been eliminated to obtain the image of the texture show in figure 4.4.3. The pixels next to the origin of the FFT image, characterized by a very low frequency, have been considered as the pixels containing information about the human face, which is the detail of the sample image analyzed in this case. The red ellipse in the picture below represents the area containing low frequencies.

Since the software used does not allow a complete and repeatable analysis, the aim of this preliminary study is just that of verifying the possibility to find specific frequencies belonging to the image impressed on the Shroud and other specific frequencies representing the texture.

As specified above in this section, with a dedicated software it is possible to have more complete and accurate results, since there is the possibility to find all the pixels of the FFT of the image which contain frequencies related to the image impressed.

The next image is the original sample, but with gray levels inverted to underline the various details appearing in the image.

Figure 4.4.1: Original Image with colors inverted to highlight details.

The two following pictures represent the image of the human face extracted from the background and the Shroud having eliminated the face impressed as much as possible, considering that Matlab does not allow a sufficiently clear plot of the frequencies found by means of the two-dimensional FFT.

This result has been gained analyzing the Fourier Spectrum of the sample image by means of a Matlab brief program and finding the pixels containing the lowest frequencies, probably corresponding to the image impressed. After the individuation of these pixels, their gray levels have been set to zero, so as to separate the texture from the image of interest.

From the results that follow, it becomes clear that the analysis proposed is merely partial, in fact the evident vertical lines crossing the eye and developing next to the nose are still clearly visible. Still, there are some horizontal lines visible which could be elimianted with a more complete analysis.

Figure 4.4.2: Image after the preliminary elaboration proposed.

Figure 4.4.3: Texture after the elimination of the greatest part of the frequencies belonging to the image, which it is possible to find using a non-dedicated software.

Another brief study done using two-dimensional Fourier transform is presented below. In this case, the other sample image presented in figure 4.3.1 has been considered: the one containing an evident spot. With the same technique applied for the previous sample image, the Fourier Spectrum of the image has been analyzed. The image texture has been found and it is visible in its different lines (directed in different ways) in the pictures below.

Figure 4.4.4: Original spot image taken from the Turin Shroud.

Figure 4.4.5: Vertical and horizontal lines in the texture.

Figure 4.4.6: Diagonal lines individuated and final image collecting all data concerning texture.

The problem with this kind of analysis is that the determination of the frequencies corresponding to vertical, horizontal and diagonal lines cannot be very precise, since the method followed to obtain the results proposed in the previous pictures proceeds by trial and error and because this brief study does not use a specific and dedicated software for spectral analysis.

This simple and preliminary work is useful, although not quantitative enough to be repeatable, because it confirms that the texture is characterized by some specific frequencies, even if it is handmade. This means that, with a more precise and appropriate study, it is possible to find these frequencies and to eliminate them.

The following chapters of this work concern a different method of analysis, based on Wavelet Functions and on their correlation with a sample signal, starting from a general approach to the various Matlab tools using Wavelet Analysis to be applied to images and signals.

5.General Description of the Shroud Image

In this chapter, a general description of the image impressed on the Shroud is given. These considerations are presented after the discussion on the various techniques of image (or signal) analysis presented in the previous chapters and before the final conclusions necessary to decide which is the most suitable procedure (among the methods proposed) to be followed to analyze the Shroud Image.

This choice is due to the necessity to consider the peculiar characteristics of the Shroud Image in order to decide which method to apply, moreover without a general introduction of the various analysis techniques, it is not possible to make the best choice.

5.1 Characteristics of the impressed image

The image impressed on the Shroud of Turin is really unique, in fact its origin has not yet been determined.

The body image is only superficial, because its formation is due to some oxidation and cellulose dehydratation process.

The adjecting parts of the body image are darker than the rest of the body, this accounts for a luminance distribution which is exactly the opposite than expected for a normal image.

Therefore, the impressed image can be compared to a photographic negative, whose spatial transposition leads to the exchange of the right with the left part of the image itself.

It must be remembered that when one is referring to the right part of the image, the left part of the body is to be accounted for, because of the spatial transposition already explained.

Another important characteristic of these images is their three-dimensionality, in fact it is possible to get some spatial information because the luminance values of the various pixels depend on the distance between the body (which impressed its image) and the towel covering it.

5.2 Main Defects belonging to the Shroud

What is to be intended when one refers to 'defects' in the Shroud images, are some signals acquired with cameras but irrelevant to the image itself.

Thus, 'defects' are:

- The herring-bone fabric;
- The folds of the fabric, due to the various and long periods in which the Shroud has been kept, which are mainly visible in the vertical and horizontal directions;
- Some horizontal defects consisting in some very thin lines;
- Vertical defects characterised by very high spatial frequencies probably following the fibres of the fabric;
- The background noise of the signal.

The frequencies characterising defects which can be found in litterature for the two images presented in figure 4.3.1, concerning vertical, horizontal and diagonal lines are reported below. In particular, it was [15] who, in his study of year 2006, analyzed the Shroud, extracting the main frequencies characterizing vertical, horizontal and diagonal bands by means of the one-dimensional wavelet transform.

The results of analysis proposed by [15] for the two images presented in figure 4.3.1 are the following:

- 1/40 , 1/16 $pixel^{-1}$ associated with vertical bands;
- 1/40, 1/160, 1/240 $pixel^{-1}$ associated with horizontal bands;
- 1/3 $pixel^{-1}$ related to the herring-bone effect of the fabric.

These results are presented with 1/pixel as the frequencies dimension, because they refer to the two sample images considered in chapter 4 for the FFT analysis example. These images are presented in figure 4.3.1.

5.3 Difficulties in finding a general method for Shroud analysis

The main difficulties concerning the choice of the most suitable method of analysis are connected with the fact that the fabric is handmade, therefore its effect on the image impressed is difficult to eliminate. It should be eliminated to be able to distinguish the details, which are otherwise confused. The problem here is that the

texture is not regular, therefore it is not easy to write a general algorithm to extract the interesting details from the image.

Another problem when thinking about frequencies is that some details have the same frequency components of the vertical or horizontal lines; thus eliminating a line can also provoque the elimination of some details.

In conclusion, a specific method has to be implemented, because the various lines are not parallel to each other and are not characterized by the same frequency.

6. Choice of the analysis method to be applied to the Shroud

In this paragraph, a brief summary of the various procedures presented is exposed. The discussion is meant to focus on every method proposed in the previous chapters, in order to understand which particular method is useful to eliminate a specific kind of defects belonging to the texture.

The two-dimensional FFT is less suitable than CWT (Continuous Wavelet Transform) to digitally analyze the Shroud. In fact, as discussed in the dedicated chapter, the Fourier Transform of a signal converts that signal in a sequence of sinusoids and cosinusoids, but the Shroud fabric is handmade, therefore it is not easy to find these components. In contrast, the Wavelet Analysis seems to be a better choice, because it is possible to adapt the wavelet function to the signal.

To be more precise, the most interesting dissimilarity between these two kinds of transforms is that individual wavelet functions are *localized in space*. Fourier sine and cosine functions are not. This localization feature, along with wavelets' localization of frequency, makes many functions and operators using wavelets "sparse" when transformed into the wavelet domain. This sparseness, in turn, results in a number of useful applications such as data compression, detecting features in images, and removing noise from time series.

One way to see the time-frequency resolution differences between the Fourier transform and the wavelet transform is to look at the basis function coverage of the time-frequency plane . Figure 6.1 shows a windowed Fourier transform, where the window is simply a square wave. The square wave window truncates the sine or cosine function to fit a window of a particular width. Because a single window is used for all frequencies in the WFT, the resolution of the analysis is the same at all locations in the time-frequency plane.

Figure 6.1: Fourier basis functions, time-frequency tiles, and coverage of the time-frequency plane.

An advantage of wavelet transforms is that the windows *vary*. In order to isolate signal discontinuities, one would like to have some very short basis functions. At the same time, in order to obtain detailed frequency analysis, one would like to have some very long basis functions. A way to achieve this is to have short high-frequency basis functions and long low-frequency ones. This medium is exactly what you get with wavelet transforms. Figure 6.2 shows the coverage in the time-frequency plane with one wavelet function, the Daubechies wavelet.

Figure 6.2: Daubechies wavelet basis functions, time-frequency tiles, and coverage of the time-frequency plane.

One thing to remember is that wavelet transforms do not have a single set of basis functions like the Fourier transform, which utilizes just the sine and cosine functions. Instead, wavelet transforms have an infinite set of possible basis functions. Thus wavelet analysis provides immediate access to information that can be obscured by other time-frequency methods such as Fourier analysis.

In general, the Huang-Hilbert Transform seems to be an analysis procedure valid to eliminate various kinds of texture defects from the Shroud, so as to highlight the image impressed. However, some more advanced studies of this new method of signal analysis are needed, before applying it. Still some mathematical problems concerning this approach need to be solved. This analysis procedure could be used instead of the Wavelet Transform method studied, because of its high efficiency and due to its high resolution, but only after the conclusions of the studies which are still in progress.

The Hough Transform could be more suitable for the most evident lines or burns which can be seen on the fabric. However, since the collocation of these lines is clearly visible, it is also possible to use some simpler methods to eliminate these defects, such as some software already available. In fact, the Hough Transform has been considered as a possible method of analysis, before finding the solution represented by the Wavelet Transform. In conclusion, this method has been presented for completeness, but there exist other alternatives which seem to be more suitable for the Shroud analysis.

After having considered the different approaches possible and keeping in mind the characteristics of the Shroud Image, the best choice seems to be that of Wavelet Analysis. The reasons for choosing wavelets is linked to their properties with respect to the image peculiarities, as highlighted above in this same section.

Another possibility could be that of Huang-Hilbert Transform, but some further mathematical study is necessary before applying it, whereas wavelets are already known and fully mathematically explained.

7. General Discussion about Defects Elimination using Wavelets

In this section some general information about wavelet application is given, with references to the advantages of this method with respect to the Fourier Transform.

The technique of wavelet transforms has obtained an important success in different scientific fields, among which image analysis. Wavelet analysis consists in comparing a mother function, suitably scaled, with the image to be analyzed.

The advantage of this technique is due to the concept of 'scale', or 'scaling', by means of which details of various size can be analyzed with adequate resolution. Moreover, the connection between scale and frequency is maintained, therefore it is always possible to find the position of the image corresponding to a specific range of frequencies.

The basic difference between Wavelet and Fourier Transforms is that in the latter, the resolution is implicitly fixed when the analysis window has been chosen.

Wavelets are considered to be interesting also for image compression, both in medical and scientific fields, where the need for a fast data flow cannot lead to inaccurate quality.

For all these reasons, the Mallat algorithm has been proposed by [14], using the Wavelet technique to eliminate the defects in some images taken from the Shroud of Turin.

In the case of these images, it is difficult to distinguish the subject from the background; this characteristic creates some difficulties in analyzing the images.
The algorithm concentrates on the elimination of the visible texture disturbing the image; in particular some vertical and horizontal lines are clearly visible an should be eliminated to underline the figure impressed on the Shroud.

This different approach to the problem, using Wavelet Analysis instead of Fourier Analysis, has been proposed because the Shroud is characterized by a series of lines, all over its length and width, which are not regular nor parallel.
They are not uniformly distributed, therefore they can be more or less visible depending on the area examined. The only characteristic they have in common is their frequency, which makes it possible to predict their presence even if they are not visible.
Wavelet Transforms give a good relation between the frequencies found and their spatial collocation, whereas using Fourier Transforms requires a continuous process of choice of the frequencies to be eliminated and control of the results obtained by means of the Inverse Transform.

The application of the wavelet technique to analyze the Turin Shroud has already been considered by [15] in the year 2006. In his work, he calculates applies the 2D wavelet analysis to digital images, considering also some images taken from the Shroud.
Anyway, this wavelet application is totally general, the work has not been thought to study specifically the sample images belonging to the Shroud of Turin.

The algorithm created in this work focuses on operating on a specific image (or some specific images), directly eliminating the defects and distinguishing among various cases.

8. Image denoise using Matlab Wavemenu

The analysis started by [14] has been completed making a comparison of their results with the ones that can be obtained using Matlab Wavemenu, and in particular the section which elaborates 2D signals. This section does not contain the final method applied to analyze the Shroud, but it is meant to focus on some Matlab tools which are available to correct images.

The various options are presented below, but they are also written in the following table, respecting the same order of the Matlab Wavemenu initial window.

Figure 8.1: Initial window for choosing the appropriate Wavelet application.

One-dimensional	**Specialized Tools 1D**
Wavelet 1D	SWT De-noising 1D
Wavelet Packet 1D	Density Estimation 1D
Continuous Wavelet 1D	Regression Estimation 1D
Complex Continuous Wavelet 1D	Wavelet Coefficients Selection
	Fractional Brownian Generation 1D
Two-dimensional	
Wavelet 2D	**Specialized Tools 2D**
Wavelet Packet 2D	SWT De-noising 2D
	Wavelet Coefficients Selection 2D
Display	Image Fusion
Wavelet Display	
Wavelet Packet Display	**Extension**
	Signal Extension
Wavelet Design	Image Extension
New Wavelet for CWT	

In this case, the option to be pushed is ‘Two-dimensional - > Wavelet 2D’. First of all, let us analyze the image of the spot, already seen in the FFT section, by means of this Matlab toolbox, which works almost automatically. To load the desired image it is sufficient to select the appropriate option in the dropdown, in this example ‘Load - > Image’:

Figure 8.2: Path to be followed to load the desired image.

The only option selected to obtain this result is ‘Analyze’, the wavelet function chosen is the Rbio6.8, with a decomposition level of 5.

Figure 8.3: Decomposition of the image at level 5 using a rbio6.8 wavelet function. Now the denoise option should be chosen to enhance the image quality.

In the image below, wavelet Rbio6.8 is presented. The acronym 'Rbio' is used to refer to Reverse Biorthogonal Wavelets.

Figure 8.4: representation of the Rbio6.8 Wavelet function.

In the following picture the left column of image 8.3 is presented. The image characteristics are in the data size, the wavelet type and also the decomposition level are visible.

Figure 8.5: Zoom of the column on the left of figure 8.3.

After having seen what image decomposition is, it is possible to make the automatic denoise of the same image. To this extent, another option has to be chosen following this path:

In this case the correct path is: ‘ Two-dimensional - > Wavelet 2D - > File - > Load Image - >De-noise’.

Figure 8.6: Path to follow to load the image for the de-noise application.

In this case the denoise has been done using a discrete Meyer wavelet (‘dmey’) illustrated with its corresponding scaling function in the picture below. The denoise option is that of a soft thresholding, with the thresholds automatically determined by Matlab for every of the five decomposition levels.

Figure 8.7: Meyer Wavelet Function and its scaling function.

Figure 8.8 shows the resulting denoise of the sample image representing the spot, using the discrete Meyer wavelet in figure 8.7. The histograms of the wavelet detail coefficients of the image on the left of the window are displayed below the image itself. These histograms are organized as follows: from the bottom for level 1 to the top for level 5. On the left there are horizontal coefficients, in the middle diagonal coefficients, and on the right vertical coefficients.

Figure 8.8: Denoised image using soft thresholding to eliminate horizontal coefficients. The wavelet used is a discrete Meyer wavelet until level 5.

Figure 8.9: Zoom of the column on the left of figure 8.8.

Figure 8.10: Denoised image using soft thresholding to eliminate diagonal coefficients, using the hsv color map. The wavelet applied is a discrete Meyer wavelet until level 5.

Figure 8.11: Denoised image using soft thresholding to eliminate vertical coefficients. The wavelet used is a discrete Meyer wavelet until level 5.

In the following pictures, the same image is analyzed using the discrete Meyer wavelet as in the previous case, but hard thresholding instead of soft thresholding.

Figure 8.12: Hard thresholding of horizontal coefficients with discrete Meyer wavelet.

Figure 8.13: Hard thresholding of diagonal coefficients with discrete Meyer wavelet.

Figure 8.14: Hard thresholding of vertical coefficients with discrete Meyer wavelet.

The same study, using the same wavelet function can be done for the other sample image considered: the human face. In this case, an example of the result that can be obtained using the Wavemenu is the following one.

Figure 8.15: Soft thresholding of horizontal coefficients with discrete Meyer wavelet.

Figure 8.16: Soft thresholding of diagonal coefficients with discrete Meyer wavelet.

Figure 8.17: Soft thresholding of vertical coefficients with discrete Meyer wavelet.

9. Results of the classical Mallat algorithm using different Mother Wavelets

In this paragraph a general summary of the results obtained by [14] for one of the two sample images studied and described in chapter 4 (figure 4.3.1) are presented. It is to be remembered that the frequencies used to eliminate image defects are the ones found by [15].

The following images have all been corrected after wavelet filtering with the Mallat pyramidal algorithm, to restore the correct gray levels.

They are interesting because they highlight the fact that results are highly dependent on the particular wavelet shape chosen to analyze the image. The vertical lines crossing the image next to the nose are always clearly visible; this means that, in their case, the problem is related to the frequencies to be eliminated and not to the most suitable wavelet (at least for the wavelet shapes analyzed).

Looking at the following pictures, one can notice the great difference among the results obtained using different wavelet functions. The reason of these differences is probably due to the different correlation degree between every specific wavelet and the image; in other words there are wavelet shapes which are more similar to the signal shape and therefore more suitable for the analysis than others.

However this conclusion can be only qualitative, because a general theory connecting the image or the signal to be analyzed to the best wavelet shape is not available.

Image 9.1 shows two results: the first on the left is the sample image after the frequencies elimination by means of the Haar wavelet, the other one on the right is the same image, but after a Daubechies 6 wavelet filtering. It is clear that the image filtered with the Db6 wavelet function keeps the original image shape better than the image filtered with the Haar wavelet.

Figure 9.1: Image elaboration with the Haar wavelet and with the Daubechies 6 wavelet.

These two images in figure 9.2 present the results obtained using the Rbio6.8 wavelet function (on the left) and the sym6 wavelet function (on the right). The image resulting from the analysis with rbio6.8 seems to have lost its initial definition more than the image analyzed with the sym6 wavelet function. As already mentioned, these effects could be the evidence that the wavelet function sym6 is more similar to the signal trend with respect to the rbio6.8 wavelet and therefore it could be more suitable to filter the signal.

However, since the frequencies eliminated do not completely cancel the texture effect behind the image impressed on the fabric, this could also be an effect accounting for the necessity to verify the appropriate frequencies to be eliminated, searching which wavelet function best resembles a signal appropriately sampled.

This last consideration will be developed more clearly in the following chapters, since it is the start of the new method of analysis proposed in this work.

Figure 9.2: Image after elaboration with a rbio6.8 wavelet and with a sym6 Wavelet.

In conclusion, from this example the following concepts should be kept in mind:

- results can change widely depending on the wavelet function selected;
- it is difficult to find the appropriate wavelet shape to be used for a particular one or two-dimensional signal and the choice process is made of several trials;
- the reason why different wavelet shapes cause so different results is probably due to the correlation of that particular wavelet with the signal;
- in the case of the sample image presented in this chapter, it is necessary to determine the frequencies to be eliminated to cancel the texture effect with a repeatable procedure.

Here are also presented the wavelet functions relative to the images proposed. In these two pictures Haar scaling function and the Haar wavelet function are presented respectively:

The following picture represents the sym 6 wavelet function and its scaling function.

Below are the Daubechies 6 wavelet function and its scaling function.

Finally, the last four images represent the Rbio6.8 wavelet functions for signal decomposition and reconstruction and the corresponding scaling functions.

10. Understanding and Interpreting Wavelet Spectrum

This paragraph is meant to focus on the interpretation of the wavelet spectrum of a signal. This interpretation is particularly difficult when the signal and noise are melted up, because it becomes very difficult to choose the correct wavelet shape. In this section some examples are presented to try and analyze the spectrum.

A sine wave and a noisy sine wave have been considered for the analysis, for these signals are very popular and widely used.

Let us start considering the simplest signal between the two, the traditional sine wave , so as to analyze its spectrum. The analysis is made using the Matlab Wavemenu. In this case the path followed is: 'One-dimensional - > Continuous Wavelet 1D - > File - > Load - > Signal'. The sine wave prepared with Matlab for this wavelet analysis has been considered in the interval between 0 and 2π and its values have been registered in a vector with a discrete step named 'i', of 0.01 rad. The period T of this sine wave is $2\pi/\omega$, and since

$$X = \sin(2i) = \sin(\omega i)$$

is the wave calculated, this means that the period is π.

The first wavelet considered is a Gaussian function with two as the wave parameter. In this case the result of this analysis is the following:

Figure 10.1: Spectrum of a sine wave using a Gaus2 Wavelet.

The signal sampled is visible as a red curve in figure 10.1 and the green curve is the signal reconstructed by means of the coefficients belonging to the scale 32. The numbers visible below these two curves refer to the data sample. In fact, the signal has been created calculating the sine wave in a certain space interval with a specific step. More precisely, the interval chosen is [0,2π] and the discrete step used is 0.01 rad. Therefore, the number below the plots is the time interval divided by the discrete step applied or, in other words, the number of points in which the function has been calculated. In the case presented:

$$\frac{2\pi}{0.01} = 628.3185,$$

which corresponds to the highest value on the right of the curves.

To have the plot the coefficients line it is sufficient to select the wavelet type and to click the option 'Analyze'. Then a plot of a coefficients line automatically selected by the software appears on the window. If one wants to move to another coefficients line, it is possible to select which line to use directly on the screen.

In this simple case one can see the two areas in the middle of the spectrum, indicating the two local maxima and minima of the signal considered. The same two peaks are also indicated by the last plot, representing local maxima of the signal wave described.

The result plotted in the previous figure refers to a Gauss2 wavelet, but here after are the results for the same signal, but using different wavelets.

Figure 10.2: Spectrum of a sine wave obtained superposing a Mexican hat to the signal.

In this case, where the Mexican hat has been applied, it is possible to verify that the local maxima or minima revealed are always the same found in the previous case. It can be concluded that both the Mexican hat and the Gaussian 2 wavelet functions are suitable to reproduce the signal. This can be also seen by looking at the plot of the coefficients line, which resembles the signal closely. In fact, the greatest differences between the original signal and the curve due to the coefficients line is visible at the ends of the plot, where there can be some deformations due to the software.

In the following picture the same signal is analyzed, but with a different mother wavelet: the Haar wavelet. In this case it is to be concluded that this wavelet shape is less suitable for reproducing the signal analyzed.

In fact, even if some maxima or minima are still detectable by means of the spectrum, they do not correspond to the ones of the initial sine wave and they are different from the ones determined with the two previous analyses. This can also be seen by looking at the shifted signal resulting from the coefficients line an plotted as a green curve.

Figure 10.3: Spectrum of the sine wave obtained using a Haar Wavelet.

This final case is that of a rbio2.4 wavelet applied to study the frequencies and the peaks of our signal. One has to conclude that this wavelet is more suitable than the previous Haar wavelet, but the signal is not reproduced accurately, as it can be seen by the green plot of the coefficients line.

Figure 10.4: Spectrum of the signal with a rbio2.4 mother wavelet.

To go further with this analysis, let us study the spectrum of a noisy sine wave already stored in the Wavemenu examples and calculated over 1000 points. Even in this case one should choose the most suitable wavelet shape.

With a Mexican hat wavelet, the result is the following:

Figure 10.5: Noisy sine wave spectrum with a Mexican hat wavelet.

Here, one can see that this spectrum is apparently more difficult to study than the spectrum of the traditional sine signal. This is because the noise present, disturbing the signal, creates some peaks of correlation at low scales. These peaks at low scales represent the noise peaks at high frequency, whereas the other peaks of correlation which can be seen by looking at the spectrum at higher scales, refer to the sine signal. In fact it is possible to see that the red spots are plotted in line with the maxima or minima of the sine wave. The green curve given automatically by the software is the signal reconstructed from the coefficients line indicated above the plot.

It is also possible to determine the sine wave frequency, selecting the coefficients line directly from the plot. The result of this analysis, choosing the coefficients line best resembling the sine trend, is a frequency of 0.006 1/m. This is an acceptable result, because the sine wave has a period of 166.67 m and the corresponding frequency is 1/T, therefore 0.006 1/m.

Figure 10.6: Noisy sine to be analyzed (red) and plot of the coefficients line chosen to determine the sine wave frequency (green).

An estimate of the noise frequency can be found repeating the same procedure, but for lowest scales.

Figure 10.7: Zoomed image representing the scales from 1 to 23, used to determine the high frequency components.

The horizontal line intersecting the spectrum is used to estimate noise frequency. The result in this case, for a scale of 3 is a frequency of 0.083 1/m. For a scale of 2, the frequency is 0.125 1/m, which is also a possible value for noise.

If another wavelet function shape is chosen, the spectrum will vary. The following example refers to the Daubechies 6 Wavelet function used to obtain the spectrum of the noisy sine wave.

Figure 10.8: Noisy sine wave spectrum with a Daubechies6 wavelet.

Since in this case, the Wavelet used is less suitable to reproduce our noisy signal, the results are less clear. There is no more the evident separation between peaks at high and low frequencies.

Some conclusions to be underlined are:

- The results of the analysis and their clearness depend on the choice of a suitable Mother Wavelet;
- The results become more difficult to interpret when noise partially or totally covers the signal;

- For time or spatially varying signals, Wavelet Spectrum gives clearer information than Fourier Spectrum;
- The correct choice in terms of wavelet function has to be determined by experience, or by looking at some similarities between the signal and the wavelet function but no specific algorithm has been developed.

11. Wavelet Analysis of two sample images of the Shroud

This chapter contains the wavelet analysis of two sample images extracted from the Shroud. The images chosen are the following ones:

Figure 11.1: Images chosen for Wavelet analysis.

The two samples already presented in chapter 4 for FFT analysis have also been chosen for the application of this study. Both the details are in gray levels and the tones conversion ratio is not known.

The picture on the left of figure 11.1 represents a spot and it has been chosen because the effect of the vertical and horizontal lines of the texture is clearly visible. This image is composed of 591 pixels in the horizontal direction and 537 pixels in the vertical direction, with a 25 pixel/cm resolution.

The picture on the right of figure 11.1 represents a human face, instead. It has been used in this study because it is considered to be one of the most interesting details to be analyzed, for which considerable improvement is desirable. This portion of the entire fabric is characterized by 661 pixels in the horizontal direction and 737 in the vertical direction, with a 25 pixel/cm resolution, as in the previous case.

Since in the previous wavelet studies it has not been possible to satisfactorily eliminate the texture defects, a new analysis procedure has been thought.

First of all, it appears to be necessary to find the frequencies corresponding to the various defects: the vertical, diagonal and horizontal lines of the fabric. Then, it is necessary to create an appropriate Mallat algorithm to decompose the image in various levels and to eliminate from each level the frequencies found in the preceding step of the analysis.

This chapter shows the method used to find the frequencies corresponding to defects and the results obtained after their elimination by means of a filtering procedure applied by means of the Mallat algorithm.

After the study of these two images, another work is proposed: the denoise of the human face without any blood traces. The method applied to all these images is always the same, but for the last image without any blood spot, the frequencies are different from the ones determined for the other two images, because the pixels numbers are different. In particular, the image analyzed after the elimination of the blood traces is the following one:

Figure11.2: Image of the human face without any blood traces.

The image in figure 11.2 is characterized by 1485x1281 pixels, where the first number represents the image matrix rows and the second number stands for the image matrix columns. The original image was a color image, but before performing the wavelet filtering it has been translated into a gray levels image, using the 'uint8' Matlab command.

11.1 Signal sampling and determination of the frequencies to be eliminated

The method chosen to analyze the images so as to obtain the frequencies corresponding to the various visible defects is the wavelet spectral analysis. This method has been applied to a one-dimensional signal extracted from the image of the spot presented in the introduction of this chapter, to determine the defects frequencies for the images of the human face and of the spot. In particular, four sample signals have been analyzed:

- Signal sampled over three horizontal lines above the spot, each composed of 591 pixels;
- Signal sampled over three vertical lines on the left of the spot, each composed of 537 pixels;
- Signal sampled over three horizontal lines below the spot, each composed of 591 pixels;
- Signal sampled over three vertical lines on the right of the spot, each composed of 537 pixels.

To be more precise, the following images show the exact location in which the samples have been taken.

Figure 11.1.1: The image on the left shows the horizontal sampling above the spot, the image on the right refers to the vertical sampling on the left of the spot.

Referring to figure 11.1.1, the horizontal sampling has been done over the pixels rows 15, 16, 17, whereas the vertical sampling over the pixels columns 3, 4, 5 to avoid the evident black line next to the left vertical border of the image, which does not represent the mean gray levels values to be attributed to the texture.

Figure 11.1.2: The image on the left shows the horizontal sampling below the spot, the image on the right refers to the vertical sampling on the right of the spot.

Referring to figure 11.1.2, the horizontal sampling has been done over the pixels rows 521, 522, 523, whereas the vertical sampling over the pixels columns 586, 587, 588.

In both cases vertical and horizontal samplings passing through the spot are avoided, for that solution would alter the mean value of the gray levels corresponding to the texture to be eliminated.

The same sampling procedure has been applied to the image of the human face without any blood trace, but since its pixels number is very different from the one of the other two sample images, new frequencies have been determined by means of another sampling. This sampling has been done exactly as the one already proposed for the other two images, but directly using the image without blood.

Below is the illustration of where the sample signals have been taken.

Figure 11.1.3: The image on the left shows the horizontal sampling below the human face, the image on the right refers to the vertical sampling on the right of the face.

Referring to figure 11.1.3, the horizontal sampling has been done over the pixels rows 1485, 1486, 1487, whereas the vertical sampling over the pixels columns 1260, 1261, 1262.

In case of horizontal sampling (gray levels of the rows of the image matrix) the final signal obtained is:

$$x = \sum_{j=1}^{n} [I(r1, j) + I(r2, j) + I(r3, j)]/3$$

where n is the number of columns in the image matrix and r1, r2, r3 are the rows selected to sample the gray levels of the texture in the horizontal direction.

Whereas, in case of vertical sampling (gray levels of the columns of the image matrix) the final signal obtained is:

$$y = \sum_{i=1}^{m} [I(i, c1) + I(i, c2) + I(i, c3)]/3$$

where m is the number of rows in the image matrix and c1, c2, c3 are the columns selected to sample the gray levels of the texture in the vertical direction.

11.1.1 How to find the frequencies to be eliminated from the one-dimensional signal sampled

As mentioned in the previous chapters, it is necessary to find the frequencies corresponding to defects. The problem is how to get this result. The idea developed is that of using the wavelet spectrum of the one-dimensional signal sampled to obtain the desired frequencies.

In particular, a continuous wavelet function has been used to plot the wavelet spectrum. From the one-dimensional signal corresponding to gray levels variations in the horizontal direction, information can be obtained about vertical defects frequencies. From the one-dimensional signal corresponding to gray levels variations in the vertical direction, one can find the horizontal defects frequencies.

The method applied in this study to find these frequencies consists in searching the correlation peaks due to the great similarity between the signal and the continuous wavelet function used to analyze it. The basis of the study is the individuation and selection of the most suitable wavelet to analyze the signal, which depends on the particular signal considered. After this first step, the continuous wavelet spectrum of the signal is plotted using Matlab Wavemenu and the final step for the frequencies search is the individuation of the correlation peaks in the spectrum.

11.2 Denoise of the spot and human face with blood images: horizontally sampled signal above the spot

This paragraph deals with the plot of the Wavelet Spectrum of the signal sampled above the spot in the horizontal direction; the sampling procedure is illustrated in figure 11.1.1. It is important to remember that from the gray levels variations in the horizontal direction, the frequencies of the vertical defects can be determined. The Wavelet Spectrum has been plotted using Matlab 'Wavemenu', which allows loading the signal to be analyzed and choosing the Wavelet function: complex continuous or simply continuous or discrete. For this analysis, a continuous wavelet spectrum (CWT) has been used.

After having chosen the Wavelet to be applied, it is also possible to choose the number of scales to be used to plot the Wavelet Spectrum.

The picture below refers to the Rbio6.8 Wavelet function and the first window represents the signal sampled along three horizontal lines, acquired as described in paragraph 11.1. The second section of the plot represents the CWT of the one-dimensional signal and below it there is a description of the correlation levels plotted in the spectrum. The last curve plotted represents the coefficients line automatically given by the software, but it is possible to select another scale to plot a different coefficients line just by clicking on the screen.

After having found the spectrum relative to the use of a specific wavelet function, the frequencies corresponding to vertical defects have to be determined. To this extent, the correlation peaks have to be found and the corresponding scales and frequencies have to be calculated.

The basic problem with this step of the method proposed concerns the correlation peaks, because the scales corresponding to the maximum correlation levels have been determined manually. In fact the scales

corresponding to the correlation peaks visible on the screen have been selected directly on the spectrum and a brief Matlab script has been used to find the corresponding frequencies.

The problem is: which correlation peaks should be considered? Only the red ones surrounded by the blue ones or is it correct to consider all the scales in which blue and red areas appear? If some correlation peaks are not aligned with some others, should they be considered to calculate the corresponding frequencies?

It has been decided to avoid considering the correlation peaks next to the vertical border lines of the spectrum, because these areas are affected by errors due to numerical effects. The areas of interest considered for the correlation peaks are the red and blue ones. Where more correlation peaks are horizontally aligned, the scales considered go from the lowest to the highest possible.

If a correlation area is not perfectly aligned with the other horizontally aligned peaks, then the process becomes still less repeatable. In fact, in this case one should decide if that correlation area could contain some relevant frequencies or not, which is not so easy.

Therefore, in this case, if the red spot surrounded by a blue ellipse is not aligned with the majority of the correlation peaks (in the horizontal direction), but it is possible to find at least one other area that shows correlation peaks for the same scales, then the corresponding frequencies are taken into account.

Since the effect to be eliminated is the texture, if some isolated correlation areas are present, they are not considered, because there is a sort of periodicity in the fabric, even if it is handmade. This has also been shown with the qualitative two-dimensional Fourier analysis of chapter four.

Figure 11.2.1: From the top: Sample Signal Acquired, Wavelet Spectrum for Scales 1:64 and coefficients line for scale 32. The wavelet function is the Rbio6.8

For this sampled signal, in the picture below, the coefficients line selection is presented. A line is visible, which crosses some correlation peaks. Since not all the areas of interest are aligned, it is not possible to find a coefficients line crossing all the correlation peaks. This is the reason of the difficulty in selecting the frequencies and this accounts for the impossibility to find a completely automatic method of analysis. Also the repeatability of the process should be improved after further studies starting from this work, because it is

the analyst who has to find a method to determine which scales have to be considered when some peaks are not aligned.

Figure 11.2.2: Selection of some peaks of correlation of the Wavelet Spectrum.

Other correlation peaks have been taken into account, to evaluate all the possible frequencies corresponding to noise. Thus, the various scales and the corresponding frequencies considered are:

Wavelet	Scales	Frequencies [1/pixel]	Frequencies [1/pixel]	Frequencies [1/pixel]	Frequencies [1/pixel]	Frequencies [1/pixel]	Frequencies [1/pixel]
Rbio6.8	5:8	0.129	0.108	0.092	0.081	/	/
Rbio6.8	13:19	0.05	0.046	0.043	0.04	0.038	0.034
Rbio6.8	20:25	0.032	0.031	0.029	0.028	0.027	0.026

11.3 Vertically sampled signal on the left of the spot

This picture shows the Wavelet Spectrum obtained by the signal sampled vertically on the left of the spot, with the procedure illustrated in figure 11.1.1. As in the previous case, three columns of pixels have been taken into account in the left part of the image, to get the sample signal. Even in this case some interesting correlation areas can be found, but they are not as evident as in the horizontally sampled signal analyzed in paragraph 11.2, in fact there are no red zones surrounded by blue areas. The best correlation peaks that can be used to find the frequencies corresponding to horizontal defects correspond to the blue areas.

Figure11.3.1: Spectral representation of the signal vertically sampled.

Following the same procedure described in the previous section for the one-dimensional signal sampled horizontally above the spot, the following frequencies have been found for the horizontal lines crossing the fabric, which should be eliminated to clean the image from the texture effects:

Wavelet	Scales	Frequencies [1/pixel]	Frequencies [1/pixel]	Frequencies [1/pixel]
Rbio6.8	19:21	0.034	0.032	0.031
Rbio6.8	22:24	0.029	0.028	0.027
Rbio6.8	25:27	0.026	0.025	0.024
Rbio6.8	28:30	0.023	0.022	0.022

11.4 Image denoise with the frequencies found on the left and above the spot

In this paragraph the results of image denoise are presented, but instead of the frequencies proposed by [15], the new frequencies are used for horizontal and vertical defects. The frequencies used to denoise the two sample images already presented have been determined with two samplings:

- The horizontal sampling above the spot, used to calculate the frequencies corresponding to vertical defects;
- The vertical sampling on the left of the spot, used to evaluate the frequencies corresponding to horizontal defects.

The algorithm used to create this result is the Mallat pyramidal algorithm for image analysis, which uses the two-dimensional signal decomposition to eliminate the defects corresponding to the frequencies determined with the two samplings described. This algorithm does not use:

1) Gray levels and contrast correction, to avoid any operation different from the simple frequency elimination;
2) Any procedure of soft or high thresholding of the image gray levels, which could be used to eliminate the herring -bone characteristic of the texture.

Only after the examination of the image resulting from the application of this filtering method, some filtering techniques have been applied, if necessary, to eliminate eventual residual texture effects of particularly high frequency.

11.4.1 Results Presentation

Here are the results for the spot image denoise, which have been found with the frequencies written in sections 11.2 and 11.3, compared with the results obtained with the horizontal and vertical frequencies determined by [15].

Figure 11.4.1.1: Results with the frequencies determined by [15], using rbio6.8, and with the new frequencies using the same wavelet function.

The results below refer to the other sample image of the Turin Shroud analyzed so far. The hypothesis made to go on with the analysis is that the frequencies of interest are the same determined for the other image containing the spot.

What is particularly important is that the vertical lines next to the nose have disappeared, having analyzed the image with the new frequency values.

Figure 11.4.1.2: The image denoised on the left, using the new vertical and horizontal frequencies determined; the image on the right shows the frequencies eliminated from the original image. The wavelet used is rbio6.8.

Figure 11.4.1.3: The same image analyzed with the frequencies calculated by [15] on the left; the right image shows the lines corresponding to the eliminated frequencies.

11.4.2 Conclusions and Comments

Comparing the results obtained using the frequencies associated with vertical and horizontal lines found in literature and the results obtained with the new frequencies found for horizontal and vertical defects, it is clear that in figure 11.4.1.2 the vertical lines passing through the eyes and next to the nose are less visible the in figure 11.4.1.3.

In figure 11.4.1.2, the image impressed on the texture is less defined than in image 11.4.1.3, but the noisy lines confusing the image with the background have been eliminated.

11.5 Signal sampled in other areas of the image of the spot

The aim of this section is to present the results of the denoise of the two sample images, using the frequencies found analyzing the signals sampled in the horizontal direction below the spot and in the vertical direction on the right of the spot. Then, a comparison of the results obtained with this second trial, with the ones exposed in paragraph 11.4 is presented.

Below are the signals sampled and the spectra of this case.

11.5.1 Horizontally sampled signal below the spot

The first spectrum presented refers to the mean gray levels variations registered in the horizontal direction below the spot. Comparing this wavelet spectrum with the one of the signal sampled in the horizontal direction above the spot, it is clear that the sampling below the spot gives different correlation peaks.

Figure 11.5.1.1: Wavelet spectrum with Rbio6.8 mother function, for a horizontally sampled signal below the spot.

Figure 11.5.1.2: Comparison between wavelet spectrum of the horizontal one-dimensional signal acquired below the spot (on the left) and the spectrum of the same signal, but acquired above the spot (on the right).

The critical frequencies resulting from the spectrum referring to the horizontal signal below the spot are resumed in the table below:

Wavelet	Scales	Frequencies [1/pixel]	Frequencies [1/pixel]	Frequencies [1/pixel]	Frequencies [1/pixel]	Frequencies [1/pixel]	Frequencies [1/pixel]
Rbio6.8	7:9	0.092	0.081	0.072	/	/	/
Rbio6.8	10:13	0.065	0.059	0.054	0.050	/	/
Rbio6.8	33:38	0.020	0.019	0.0182	0.018	0.018	0.017

11.5.2 Vertically sampled signal on the right of the spot

In this case, three different pixel columns on the right of the spot have been analyzed, finally calculating the average value of the gray levels, as done in the previous other cases. The picture below shows the resulting wavelet spectrum of the signal obtained, whereas the frequencies written in the table have been determined by analyzing the wavelet spectrum to find the best correlation areas and the corresponding scales.

After having found the scales where the correlation peaks are present, it is possible to know the corresponding frequencies using an appropriate Matlab function or selecting the coefficients line of a specific scale directly on the window containing the spectrum. In fact the software specifies the scale and the frequency corresponding to the coefficients line selected.

Figure 11.5.2.1: Wavelet spectrum of the signal sampled in the vertical direction on the right of the spot and zoom in of the coefficients line optional plot.

Wavelet	Scales	Frequencies [1/pixel]	Frequencies [1/pixel]	Frequencies [1/pixel]	Frequencies [1/pixel]	Frequencies [1/pixel]
Rbio6.8	9:13	0.072	0.065	0.059	0.054	0.05
Rbio6.8	14:20	0.046	0.043	0.04	0.038	0.034
Rbio6.8	21:23	0.032	0.029	0.028	/	/

Even in the case of the vertical gray levels variations, the correlation peaks found with the two one-dimensional vertical signals sampled on the right or left of the spot are different. The following figure is meant to underline these differences, because it plots both the spectrum of the signal acquired on the left and the one acquired on the right of the spot.

Figure 11.5.2.2: Wavelet spectrum of the signal sampled on the right of the spot (on the left) and on the left of the spot (on the right).

These two images highlight the fact that the peaks of correlation corresponding to the two samplings are not only different, but with the vertical sampling on the right of the spot, it is possible to obtain higher correlation peaks, because red areas are present in a great number than in the spectrum of the other vertical sampling.

In conclusion, one should expect a more efficient denoise of the sample images in the horizontal direction, when using frequencies determined with the vertical sampling of gray levels variations on the right of the spot.

11.6 Image denoise using the frequencies found on the right and below the spot

In this chapter, a comparison is made between the frequencies found in the previous chapters for horizontal and vertical defects. The comparison is discussed using the denoised images by means of a Rbio6.8 Wavelet function. The problem is to choose which of the two horizontal and of the two vertical one-dimensional samplings is the more effective to denoise the image eliminating the texture effects.

Let us resume the interesting frequencies found in the horizontal and vertical directions. The location of the sampling is specified above each table.

Above the spot:

Wavelet	Scales	Frequencies [1/pixel]	Frequencies [1/pixel]	Frequencies [1/pixel]	Frequencies [1/pixel]	Frequencies [1/pixel]	Frequencies [1/pixel]
Rbio6.8	5:8	0.129	0.108	0.092	0.081	/	/
Rbio6.8	13:19	0.050	0.046	0.043	0.04	0.038	0.034
Rbio6.8	20:25	0.032	0.031	0.029	0.028	0.027	0.026

Below the spot:

Wavelet	Scales	Frequencies [1/pixel]	Frequencies [1/pixel]	Frequencies [1/pixel]	Frequencies [1/pixel]	Frequencies [1/pixel]	Frequencies [1/pixel]
Rbio6.8	7:9	0.092	0.081	0.072	/	/	/
Rbio6.8	10:13	0.065	0.059	0.054	0.050	/	/
Rbio6.8	33:38	0.020	0.019	0.0182	0.018	0.018	0.017

On the left:

Wavelet	Scales	Frequencies [1/pixel]	Frequencies [1/pixel]	Frequencies [1/pixel]
Rbio6.8	19:21	0.034	0.032	0.031
Rbio6.8	22:24	0.029	0.028	0.027
Rbio6.8	25:27	0.026	0.025	0.024
Rbio6.8	28:30	0.023	0.022	0.022

On the right:

Wavelet	Scales	Frequencies [1/pixel]	Frequencies [1/pixel]	Frequencies [1/pixel]	Frequencies [1/pixel]	Frequencies [1/pixel]
Rbio6.8	9:13	0.072	0.065	0.059	0.054	0.05
Rbio6.8	14:20	0.046	0.043	0.04	0.038	0.034
Rbio6.8	21:23	0.032	0.029	0.028	/	/

The differences in the frequencies found in these two samplings, is due to the fact that the texture is hand-made, therefore fibers are not regularly disposed in it.

This result should have been expected, considering the origin of the fabric. Since the frequencies of the two analyses are of the same order of magnitude, the accuracy of the study is confirmed.

Let us use the frequencies found with the vertical sampling on the right of the spot and below it, to denoise the sample images and make a comparison with the results obtained with the other two samplings, which are exposed in paragraph 11.4.1.The frequencies highlighted in red are coincident in the two samplings following the same direction (horizontal or vertical).

Figure 11.6.1: On the left, result obtained eliminating the frequencies of the right and below samplings, on the right result after the elimination of the frequencies found from the left and above samplings.

Figure 11.6.2: On the left, result obtained eliminating the frequencies of the right and below samplings, on the right result after the elimination of the frequencies found from the left and above samplings.

Examining the results obtained for the two sample images of the spot and of the human face, it is possible to conclude that the horizontal sampling taken above the spot gives a better result for the elimination of the frequencies representing vertical defects with respect to the signal sampled below the spot; whereas the vertical sampling on the left of the spot allows the elimination of more horizontal defects than the sampling taken on the other side of the spot. This can be seen more clearly seen by looking at the following zooms:

Figure 11.6.3: Zoom of the images presented in figure 11.6.1. On the left, vertical lines are more visible than in the other image, whereas in the image on the right, horizontal gray levels variations are more evident than in the other image.

Figure 11.6.4: Zoom of the images presented in figure 11.6.2. On the left, vertical lines are more visible than in the other image, whereas in the image on the right, horizontal gray levels variations are more evident than in the other image.

11.6.1 Conclusions and Comments

With the horizontal sampling below the spot, vertical lines are more visible than in the previous analysis corresponding to the sampling in the horizontal direction above the spot. In contrast, considering the two

vertical samplings, the second case presented of the sampling on the right of the spot is the more effective to eliminate the horizontal defects. This last consideration could also have been expected, since the analysis of the wavelet spectra of the two vertical samplings show higher correlation peaks and clearer correlation areas in the case of the sampling on the right of the spot.

It must be said, however, that the frequencies found in both the cases analyzed are more effective in eliminating defects due to texture, than those already present in literature.

In conclusion, it seems appropriate to consider the frequencies determined with the horizontal sampling above the spot and the frequencies determined with the vertical sampling on the right of the spot, to denoise respectively the vertical and horizontal defects of the sample images considered so far.

11.7 Image best denoise using the frequencies found on the right and above the spot

Here the analysis is applied to the two sample images using the frequencies corresponding to the horizontal sampling above the spot and the ones corresponding to the vertical sampling on the right of the spot.

Therefore, the frequencies used are:

Above:

Wavelet	Scales	Frequencies [1/pixel]	Frequencies [1/pixel]	Frequencies [1/pixel]	Frequencies [1/pixel]	Frequencies [1/pixel]	Frequencies [1/pixel]
Rbio6.8	5:8	0.129	0.108	0.092	0.081	/	/
Rbio6.8	13:19	0.050	0.046	0.043	0.04	0.038	0.034
Rbio6.8	20:25	0.032	0.031	0.029	0.028	0.027	0.026

On the right:

Wavelet	Scales	Frequencies [1/pixel]	Frequencies [1/pixel]	Frequencies [1/pixel]	Frequencies [1/pixel]	Frequencies [1/pixel]
Rbio6.8	9:13	0.072	0.065	0.059	0.054	0.05
Rbio6.8	14:20	0.046	0.043	0.04	0.038	0.034
Rbio6.8	21:23	0.032	0.029	0.028	/	/

The following pictures show the results obtained for the two sample images using the frequencies in the tables above.

Figure 11.7.1: Results for best vertical and horizontal defects elimination.

Figure 11.7.2: Previous results, with worse horizontal (on the left)or vertical (on the right) defects elimination, with respect to the denoise presented in figure 11.7.1.

Figure 11.7.2 shows the results of the denoise after the elimination of the frequencies corresponding to the samplings on the right and below the spot, on the left; on the right the results of the denoise using the frequencies found with the samplings on the left and above the spot are presented.

The following images are the zooms of the three results presented in figure 11.7.1 and 11.7.1. Their aim is to highlight the residual defects in the results presented in figure 11.7.2, which have been eliminated considering the frequencies extracted from the signals acquired on the right and above the spot.

In conclusion, starting from the left, the following image presents the results obtained respectively with the samplings:

- On the right and below the spot;
- On the left and above the spot;
- On the right and above the spot.

Examining the denoised images, it is clear that the best result is represented by the third set of frequencies considered and written in the two tables at the beginning of this paragraph.

Figure 11.7.3: Denoised images obtained applying the different set of frequencies corresponding to the three samplings mentioned above. The first on the left shows evident vertical defects, the one in the middle shows horizontal gray levels variations, the one on the right shows the best combination of the two previous results.

Considering the same frequencies used for the sample image of the human face and using them to denoise the sample image of the spot, the results obtained are the following ones:

Figure 11.7.4: Results for best vertical and horizontal defects elimination.

In conclusion, starting from the left, the following image presents the results obtained respectively with the samplings:

- On the right and below the spot;
- On the left and above the spot;
- On the right and above the spot.

Examining the denoised images, it is clear that the best result is represented by the third set of frequencies considered and written in the two tables at the beginning of this paragraph.

Figure 11.7.5: Denoised images obtained applying the different set of frequencies corresponding to the three samplings mentioned above. The first on the left shows evident vertical defects, the one in the middle shows horizontal gray levels variations, the one on the right shows the best combination of the two previous results.

11.8 Frequencies found from the samplings above and on the right of the spot and corresponding defects

It has already been said that the more effective sampling between the first and the second, was the first, presented in section 11.7.

Thus, in this other paragraph a correspondence between the defects and the specific frequencies considered is searched.

In this analysis a verification has to be done among the frequencies in the tables below (the first referring to vertical lines and the second to horizontal defects) , to see which frequency is effectively corresponding to which defect of the image considered.

Frequencies from the horizontal sampling, corresponding to possible vertical defects:

Wavelet	Scales	Frequencies [1/pixel]	Frequencies [1/pixel]	Frequencies [1/pixel]	Frequencies [1/pixel]	Frequencies [1/pixel]	Frequencies [1/pixel]
Rbio6.8	5:8	0.129	0.108	0.092	0.081	/	/
Rbio6.8	13:19	0.050	0.046	0.043	0.04	0.038	0.034
Rbio6.8	20:25	0.032	0.031	0.029	0.028	0.027	0.026

Frequencies from the vertical sampling, corresponding to possible horizontal defects:

Wavelet	Scales	Frequencies [1/pixel]	Frequencies [1/pixel]	Frequencies [1/pixel]	Frequencies [1/pixel]	Frequencies [1/pixel]
Rbio6.8	9:13	0.072	0.065	0.059	0.054	0.05
Rbio6.8	14:20	0.046	0.043	0.04	0.038	0.034
Rbio6.8	21:23	0.032	0.029	0.028	/	/

11.8.1 Verification of Vertical Defects

After some trials, it has been found that frequencies 0.108, 0.081, 0.05, 0.043, 0.04, 0.038, 0.034, 0.032, 0.031, 0.029, 0.028 and 0.027 1/pixel are not necessary to denoise the image of the face, because they do not correspond to vertical lines.

Whereas frequency 0.092 1/pixel cannot has to be eliminated to obtain good results. The following image shows the results eliminating this frequency, to demonstrate the necessity of its elimination.

Figure 11.8.1.1: Result obtained eliminating frequency 0.092 1/pixel and not eliminating it.

The evidence of the need to eliminate also frequency 0.129 1/pixel, is visible from the following comparison of images:

Figure 11.8.1.2: Result obtained eliminating frequency 0.129 1/pixel and the others for vertical defects and not eliminating it.

In effect, this necessity still becomes more clear by looking at the following zoomed images.

Figure 11.8.1.3: Result eliminating all the frequencies corresponding to vertical defects but 0.129 1/pixel.

Figure 11.8.1.4: Resulting denoised image eliminating frequency 0.129 1/pixel and all the other frequencies corresponding to vertical defects.

From these pictures, one can seen that both frequencies 0.129 and 0.092 1/pixel are responsible for some of the vertical lines visible next to the nose and crossing the eye, therefore they should be eliminated to obtain a satisfactory result.

Going on with this analysis considering frequency 0.046 1/pixel, it also appears necessary to eliminate its contribute and this can be easily concluded examining the following results.

Figure 11.8.1.5: Result obtained eliminating frequency 0.046 1/pixel and the others for vertical defects and not eliminating it.

Finally, considering frequency 0.026 1/pixel, the following comparison of results is considered as the evidence of the need to eliminate it.

Figure 11.8.1.6: Result obtained eliminating frequency 0.026 1/pixel and the others for vertical defects and not eliminating it.

The same analysis has been applied to the spot image, to determine which frequencies really correspond to vertical defects and to optimize results.

Figure 11.8.1.7: On the left, the result eliminating only the frequencies of the vertical defects found to be necessary, on the right the result eliminating all the frequencies but 0.129 1/pixel.

Figure 11.8.1.8: On the left, the result eliminating the frequencies of the vertical defects but 0.092 1/pixel , on the right the result eliminating all the frequencies but 0.046 1/pixel.

Figure 11.8.1.9: Resulting denoised image after the elimination of all the frequencies of the vertical defects but 0.026 1/pixel.

From the results presented, it is possible to conclude that the frequencies corresponding to vertical defects, found with the horizontal sampling above the spot, which necessarily have to be eliminated to denoise the two sample images considered in this study are:

Wavelet	Scales	Frequencies [1/pixel]
Rbio6.8	5	0.129
Rbio6.8	7	0.092
Rbio6.8	14	0.046
Rbio6.8	25	0.026

11.8.2 Verification of horizontal defects

The same procedure followed in the previous paragraph is proposed here to analyze the frequencies corresponding to horizontal defects.

After some trials, it has been found that the frequencies 0.032, 0.38, 0.04, 0.043, 0.046, 0.05, 0.054, 0.059, 0.065 1/pixel do not correspond to horizontal defects present in the spot image.

Whereas, frequency 0.034 1/pixel should be eliminated to obtain satisfactory results. As an evidence of this conclusion, the following pictures compare the results obtained eliminating this frequency and those obtained not eliminating it.

Figure 11.8.2.1: Result obtained eliminating frequency 0.034 1/pixel and the others accounting for horizontal defects and not eliminating it.

Repeating the same analysis for the frequency 0.028 1/pixel, it comes out that horizontal defects characterized by this frequency should be eliminated to effectively denoise the image. This conclusion is supported by the next image.

Figure 11.8.2.2: Result obtained eliminating frequency 0.028 1/pixel and the others for horizontal defects and not eliminating it.

Repeating the same analysis for the frequency 0.072 1/pixel, it comes out that horizontal defects characterized by this frequency should be eliminated to effectively denoise the image. This conclusion is supported by the next image.

Figure 11.8.2.3: Result obtained eliminating frequency 0.072 1/pixel and the others for horizontal defects and not eliminating it.

The same analysis has been applied to the other sample image, to determine which frequencies really correspond to horizontal defects, so as to attribute to each defect its characteristic frequency.

Figure 11.8.2.4: On the left, the result eliminating all the frequencies of the horizontal defects found to be necessary, on the right the result eliminating all the frequencies but 0.028 1/pixel.

Figure 11.8.2.5: On the left, the result eliminating all the frequencies but 0.034 1/pixel; on the right the result eliminating all the frequencies corresponding to horizontal defects but 0.072 1/pixel.

From the results, it is possible to conclude that the frequencies corresponding to vertical defects, which have necessarily to be eliminated to denoise the two images considered in this study are:

Wavelet	Scales	Frequencies [1/pixel]
Rbio6.8	19	0.028
Rbio6.8	23	0.034
Rbio6.8	9	0.072

The results obtained and proposed so far have been obtained using no soft nor hard thresholding techniques to denoise the high frequency components due to residual texture effects; no color correction nor contrast enhancement techniques have been used.

11.8.3 Verification of diagonal defects

In this particular section, the need to eliminate diagonal defects frequencies is verified. By looking at the first two pictures proposed below, it seems to be unnecessary to eliminate the frequency 1/3 1/pixel. In fact this elimination apparently does not cause any changes concerning diagonal texture, in the original image.

The fact that this conclusion is wrong becomes evident while analyzing the following two zoomed images.

Figure 11.8.3.1: Resulting image eliminating the frequency 1/3 1/pixel and without its elimination.

Figure 11.8.3.2: Zoomed Image resulting from the elimination of 1/3 1/pixel frequency.

Figure 11.8.3.3: Zoomed Image without eliminating 1/3 1/pixel frequency.

From these two pictures, it can be concluded that 1/3 1/pixel is an appropriate frequency value to be assigned to diagonal defects of the texture, even if some traces of the texture are still present.

11.9 Filtering techniques to eliminate the residual texture effects

In the previous chapters and paragraphs, the best results obtained have been those relative to the frequencies found analyzing the signal sampled above the spot in the horizontal direction and the signal sampled vertically on the right of the spot.

The results for the two sample images analyzed so far are as follows:

Figure 11.9.1: Images after the elimination of the frequencies corresponding to vertical and horizontal defects.

As it can be seen clearly, some texture effects are still present on the fabric, especially in the diagonal direction (45° from the horizontal direction). After the elimination of these residual effects, the results would be even better than the ones obtained so far.

Let us apply the following filtering techniques:

- The Median Filtering;
- The Gaussian Filtering.

The pictures that follow represent the resulting denoised image of the human face after the only wavelet filtering (11.9.2), after the wavelet and median filtering processes (11.9.3) and after the wavelet and Gaussian filtering methods (11.9.4).

Figure 11.9.2: Denoised image of the human face zoomed in.

Figure 11.9.3: Denoised image of the human face after median filtering zoomed in.

Figure 11.9.4: Denoised image of the human face after Gaussian filtering with radius 1,5, zoomed in.

The same comparison can be done to analyze the image of the spot; the quality of the denoise improves for both the sample images of the human face and of the spot, if other filtering techniques are applied after wavelet filtering.

The pictures that follow represent the resulting denoised image of the spot after the only wavelet filtering (11.9.5), after the wavelet and median filtering processes (11.9.6) and after the wavelet and Gaussian filtering methods (11.9.7).

Figure 11.9.5: Denoised image of the spot zoomed in.

Figure 11.9.6: Denoised image of the human face after median filtering zoomed in.

Figure 11.9.7: Denoised image of the spot after Gaussian filtering with radius 1,5 zoomed in.

In both the sample images, an improvement of the image quality can be seen after the median filter application, in fact less of the residual high frequency components are noticeable after the filter application. The same is true also for the images obtained after the Gaussian filtering with a radius of 1,5.

11.10 Denoise of the human face without any blood trace

The method followed to denoise the image of the human face without blood traces is the same applied to the two images described so far, thus the operations made to analyzed this image have been:

- Enhance gray levels contrasts to have an image with clearer details and to make the successive operations to plot the wavelet spectrum easier;
- Take a vertical and a horizontal sampling of the gray levels variations over the texture;
- Plot the respective wavelet spectra;
- Analyze them to obtain the frequencies corresponding to vertical and horizontal defects;
- Eliminate those frequencies to obtain the denoised image;
- Verify the possibility to improve the results with the application of other filtering techniques.

The image before and after the gray level and contrasts corrections are presented below:

Figure 11.10.1: Image to be analyzed before and after contrasts enhancement.

The horizontal and vertical samplings done over the image with enhanced contrasts lead respectively to the following spectra, obtained with the Rbio6.8 wavelet function.

Figure 11.10.2: Wavelet Spectra for the horizontal sampling (left) and for vertical sampling (right). The red curves represent the signals sampled.

At this point, it has been possible to isolate the most interesting scales and the corresponding frequencies to eliminate defects. The method applied to choose which correlation areas should be considered is exactly the same proposed and fully described in paragraph 11.2.

Thus, the frequencies which, after various trials, proved to be related to defects, are resumed in the following tables:

Vertical defects (from the horizontal sampling):

Wavelet	Scales	Frequencies [1/pixel]
Rbio6.8	47	0.014
Rbio6.8	23	0.028
Rbio6.8	18	0.036
Rbio6.8	10	0.065
Rbio6.8	3	0.216

Horizontal defects (from the vertical sampling):

Wavelet	Scales	Frequencies [1/pixel]
Rbio6.8	41	0.016
Rbio6.8	14	0.046
Rbio6.8	10	0.065
Rbio6.8	5	0.129

The resulting denoised image, after the application of the Mallat algorithm to eliminate the frequencies corresponding to vertical and horizontal defects is presented below.

Figure 11.10.3: Image of the human face without blood traces after frequencies elimination. In the middle a zoom of the same image which highlights the residual diagonal lines due to the filtering process. On the right, the defects eliminated.

Figure10.11.4: Image revealing the residual diagonal texture effect and the diagonal lines appeared because of the filtering process.

Since some diagonal lines due to the filtering process and the herring-bone texture effect are still present, it is possible to apply some other filtering technique to try and improve what has just been obtained. As an example, the two following pictures refer to the application of a Gaussian filter of radius 2.

Figure 11.10.5: Human face image after the application of a Gaussian filter of radius 2.

By looking at the zoom in figure 11.10.5, it appears clear that the Gaussian filtering process with radius 2 has improved image quality, since the diagonal residual texture effects have disappeared. There still are some

diagonal gray level variations due to the wavelet filtering process, which could be eliminated to go further with the image denoise.

To have an idea of some of the frequencies eliminated and the corresponding defects, it is possible to have a look at the following images. The first starting from the left shows the resulting denoised image without eliminating the frequency 0.016 1/pixel corresponding to horizontal defects, the one in the middle shows the result without the elimination of the frequency 0.028 1/pixel, referring to vertical defects. Finally, the picture on the right represents the denoised image without eliminating the frequency 0.036 1/pixel.

Figure 11.10.6: On the left, result not eliminating the frequency 0.016 1/pixel for horizontal defects, in the middle result not eliminating the frequency 0.028 1/pixel for vertical defects, on the right result without the elimination of the frequency 0.036 1/pixel.

12. Final Conclusions of the Study

In the previous chapters the complete method applied for the frequency analysis has been explained and exposed. The basic steps followed are:

1- Extract a sample signal from the image in the horizontal direction (i.e. create a vector containing the gray levels of the pixels corresponding to a specific row of the image matrix). To avoid coarse errors, it is recommended to consider, for example, three different rows of pixels and to make the average values of the gray levels of the corresponding pixels.

2- Define the most suitable wavelet shape and type, to create the Wavelet Spectrum of the signal using Matlab Wavemenu. For this step of the process applied, no specific approaches are defined, because there is lack of algorithms capable of individuating the wavelet which best fits to a particular signal.

3- Analyze the Spectrum to find the correlation peaks. This analysis has been done observing the resulting signal Spectrum and searching the correlation peaks frequently repeated along the signal length.

4- Decide which scales probably contain defects and find the corresponding frequencies. In this case, Matlab allows using a function called 'scal2frq' , which converts the scales individuated by the spectrum analysis into the corresponding frequencies.

5- Use the general Mallat algorithm , with the new frequencies found from the analysis presented in this work, to see if image denoise improves. At this point, if an enhancement of the denoise effectiveness is evident, it is possible to verify which of the frequencies found correspond to a specific defect and which do not, to find the final set of effective frequencies. Otherwise, the Wavelet Spectrum of the signal probably has not been analyzed correctly and it reveals to be necessary to go back to steps 3-4.

6- Final comparison between the image denoise with the two different sets of frequencies: those proposed by [15] and the ones found in this study.

7- Eventual application of other filtering techniques to eliminate the highest texture frequencies remaining after wavelet filtering.

The most important differences between this analysis and the one done by [14] are as follows:

- No color correction is applied to the resulting image after the elimination of the frequencies connected with defects;
- No enhancing contrasts technique is applied to the image;
- The dark and thin lines next to the nose and the ones passing through the eye have been eliminated in one of the images considered;
- The dark line on the left of the spot has almost been eliminated, without using any other process than finding the frequencies connected with vertical and horizontal defects;
- New frequencies corresponding to vertical and horizontal defects have been determined by means of the one-dimensional signals sampled and of the wavelet spectrum.

Here is the final comparison of the previous results obtained by [14] with the ones of this study for both the images considered:

Figure 12.1: Original image.

Figure 12.2: Results of wavelet filtering proposed by [14] using the frequencies calculated by [15].

Figure 12.3: Final results of this study, using the frequencies found and the median filtering.

Figure 12.4: Final results of this study, using the frequencies found and the Gaussian filtering with a radius of 1,5.

Figure 12.5: Wavelet Coefficients for the various details after image denoise with the frequencies found by [15].

Figure 12.6: Wavelet Coefficients for the various details after image denoise with the frequencies found in this study.

The same comparison is done for the image of the spot:

Figure 12.7: Resulting spot image after the elimination of the frequencies determined by [15].

Figure 12.8: Zoom of image 12.7.

Figure 12.9: Resulting spot image after the elimination of the frequencies found in this study.

Figure 12.10: Zoom of image 12.8.

Figure 12.10: Wavelet Coefficients for the various details after image denoise corresponding to images 12.7 and 12.9.

Figure 12.11: Wavelet Coefficients for the various details after image denoise with the frequencies found by [15].

13. Analysis of the entire Shroud

In this chapter the whole Shroud is examined. Since Matlab has some problems in dealing with memory, the method applied starts by dividing the Shroud into two parts: the front and the back. Then, a new subdivision reveals to be necessary and every part is divided once again into three different images.

At this point it is possible to analyze a sample signal, extracted from an image belonging to the front of the Shroud, to get the frequencies corresponding to the vertical lines crossing the image impressed.

With the hypothesis that the frequencies extracted from the sample signal (with exactly the same procedure described in the previous paragraphs for the other sample images of the spot and of the human face) are the same appearing in the other images, it is possible to try and clean the entire fabric.

13.1 Analysis of the front of the Shroud

In this chapter the technique used to analyze the entire front of the Turin Shroud is discussed. The original complete image is characterized by the following dimensions: 12090 x 3498 pixels, considering both the front and back sides.

The program used to eliminate defects and to highlight the image from the background is Matlab, as for the previous images treated.

In this case, the problem is related to the incapability of this software to work with an image of such dimensions, in fact the massage appearing on the command window for this kind of troubles is ‘out of memory-error unknown’.

Therefore, a way has to be found to analyze the image bypassing this memory problem. The idea developed is very simple: the original image has been divided into three images, each with the same number of columns. The three sub-images obtained have finally been recomposed (after the frequency analysis) to create the Shroud image after correction.

These are the images used:

Figure 13.1.1: The next table refers to these images numbered as 1,2,3 from the left.

The characteristics corresponding to each of the previous images are as follows:

Image number	Image Dimensions	Coordinates with respect to the original
1	1500x901	(6500:7999,1200:2100)
2	2000x901	(8000:9999,1200:2100)
3	2001x901	(10000:12000,1200:2100)

The most evident defects of the texture are some vertical lines, as seen by the images already analyzed; therefore, first of all, the same spectral analysis using wavelet continuous functions already explained in the previous chapters has been done.

This study requires the use of Matlab Wavemenu and the same procedure described in chapter eleven is adopted. In this particular case, the analysis has to be repeated because the image of the complete Shroud is composed by a greater number of pixels in comparison with those already seen, thus the frequency of the vertical and horizontal lines is not the same previously found.

The horizontal signal has been sampled using a sub-image of trial, containing a detail to control the results of eliminating some frequencies. This sample image is the sub-image containing the hands. The procedure followed to extract this signal is exactly the same seen for the other images.

The choice of the sub-image representing the hands is due to the possibility of taking a sample of the gray levels below the hands, in order not to consider eventual defects like spots, which could lead to errors in assessing signal and which are present in other areas.

Another reason for this choice is the fact that this sub-image has the hands as the most important detail, therefore it is possible to take a sample signal using some pixels which do not cross the detail itself. This would not have been possible with other sub-images, where the details occupy more or less the entire image.

However, this is only a matter of choice, since there is no scientific evidence of the impossibility to proceed in a different way. As already said, the presence of a detail to be preserved (in this case the hands) in the

sample image, is necessary to evaluate the real need to eliminate some frequencies and to make a rapid verification.

Figure 13.1.2: Image used as a sample to find the frequencies to be eliminated from the entire Shroud.

After several trials, the best correlation in the spectral analysis seemed to be found by using the Rbio6.8 wavelet function. The corresponding spectrum is presented below:

This spectrum is more difficult to interpret than the ones previously analyzed for the other sample images, because the contrast of the various gray levels have not been improved, yet. The original image has simply been traduced in its corresponding gray levels, without any other modification.

The scales and the corresponding frequencies which could correspond to anomalies are:

Wavelet	Scales	Frequencies [1/pixel]	Frequencies [1/pixel]	Frequencies [1/pixel]	Frequencies [1/pixel]	Frequencies [1/pixel]
Rbio6.8	8:11	0.081	0.072	0.065	0.059	/
Rbio6.8	14:18	0.046	0.043	0.04	0.038	0.036
Rbio6.8	19:22,28	0.034	0.032	0.031	0.029	0.023

Instead of using the procedure explained for the other images analyzed, here the maximum correlation peaks are green, which means that the maximum possible correlation (red area) is not present. Therefore the areas used for the analysis are the green ones, which represent in this spectrum the highest correlation level.

After some trials to verify which frequencies really correspond to vertical defects, the effective frequencies have resulted to be (the process used is the same described in the previous paragraphs for the two sample images of the spot and of the human face):

Wavelet	Scales	Frequencies [1/pixel]
Rbio6.8	11	0.059
Rbio6.8	18	0.036
Rbio6.8	28	0.023

The resulting image after the elimination of these frequencies corresponding to vertical crossing lines, compared with the original corresponding sub-image is presented below:

Figure 13.1.3: Sample image before and after elaboration.

Figure 13.1.4: Zoom of the sample image before frequency elimination.

Figure 13.1.5: Zoom of the sample image after frequency elimination.

The most interesting result for the frequencies corresponding to vertical defects concerns the vertical lines passing through the hands. From the following picture it is clear that the frequency corresponding to the them is 0.023 1/pixel, since they do not disappear if that frequency component is not eliminated.

Figure 13.1.6: Result obtained eliminating the frequencies 0.059 and 0.036 1/pixel but not eliminating 0.023 1/pixel.

After having verified that the elimination of the frequencies written in the previous table improves the sample image quality, the hypothesis done to proceed with this study is that the same frequencies found in the hands image are related to defects also in the other sub-images in which the Shroud has been divided.

Using this hypothesis, the various sub-images have been analyzed separately to eliminate the frequencies said. The results have been collected and then put together to create the final image free from the most evident vertical lines.

From the following pictures it is possible to observe the results of this operation; in particular the third image on the right has been obtained inverting the original gray levels to highlight the image impressed on the Shroud. If compared with other techniques, for example based on a Fourier spectrum analysis, it is clear that the method applied in this study maintains the details in a better way, avoiding to excessively smooth the impressed image.

Figure 13.1.7 : From the left: original image, elaborated image with vertical and diagonal frequencies eliminated and same image with colors inverted. The last image on the right represents the vertical lines eliminated from the original image.

The following image represents the front side of the Shroud, but in this case the original image has been personally elaborated after the gray levels conversion. In fact the original image is a color image in which the most interesting colors are red, corresponding to blood spots left on the fabric and yellow, corresponding to fibers.

Therefore, to improve the definition of the image superficially impressed on the Shroud, the gray levels corresponding to yellow and red have been changed to obtain a better result than it is possible to have from an automatic conversion.

The software used is 'Adobe Photoshop CS3 Extended, version 10.0.1', which allows the operations described.

The path followed is: 'immagine -> regolazioni -> bianco e nero'. At this point the best levels were chosen to highlight the details of the original image to be preserved. The values chosen are: -105 for red details and -160 for the yellow ones to become clearer.

Figure 13.1.8: Front side of the Shroud after gray levels elaboration.

The analysis done so far has been repeated for the image obtained this way, because the details are more defined than in the original image converted to gray levels. The result of vertical lines elimination is the following:

The results obtained can be further analyzed, remembering that no frequencies corresponding to horizontal defects have been eliminated.

Therefore another sample sub-image has been used to find the frequencies that could correspond to horizontal defects. The image chosen in this case is the one containing the legs and this choice is due to the evident horizontal lines crossing it.

Therefore, a signal has been sampled and its average value, calculated over three pixel columns with the procedure described in chapter eleven, has been considered. Its wavelet spectrum has been found using the Rbio6.8 wavelet function available in Matlab Wavemenu. As already seen for vertical frequencies, the resulting frequencies corresponding to the correlation peaks, have also been used to clean the other two sub-images.

Here is the wavelet spectrum and the signal sampled represented as a red curve:

Figure 13.1.9: Wavelet spectrum of the signal sampled over three columns of the legs image.

In the following table the frequencies of possible horizontal defects and their corresponding scales are presented:

Wavelet	Scales	Freq. [1/pixel]	Freq. [1/pixel]	Freq. [1/pixel]	Freq. [1/pixel]	Freq. [1/pixel]	Freq. [1/pixel]	Freq. [1/pixel]	Freq. [1/pixel]	Freq. [1/pixel]
Rbio6.8	16:24	0.04	0.038	0.036	0.034	0.032	0.031	0.029	0.028	0.027
Rbio6.8	25:33	0.026	0.025	0.024	0.023	0.022	0.022	0.021	0.02	0.02

After a series of trials, it has been possible to determine which frequencies do really correspond to horizontal defects. Thus, the frequencies and the corresponding scales representing defects among the ones found from the spectrum are:

Wavelet	Scales	Freq. [1/pixel]
Rbio6.8	19	0.034
Rbio6.8	28	0.023

The following images show the results obtained; it is clear that the elimination of the frequencies corresponding to the horizontal lines crossing the image leads to a smoothing of the image, which reveals less clear details.

Figure 13.1.10: Resulting image not eliminating frequencies corresponding to horizontal defects (on the left) and eliminating them. The third image on the right shows the vertical and horizontal lines eliminated from the original image for the central image.

The following images represent the results in figure 13.1.10, after the enhancement of the luminance and contrasts, to observe the most important details of the frontal image.

Figure 13.1.11: Zoomed pictures taken from the complete image of the Shroud after vertical and horizontal lines elimination.

Here are the images proving the need to eliminate the frequencies 0.034 and 0.023 1/pixel from the sub-image representing the legs.

Figure 13.1.12: From the left: horizontal defects denoise without eliminating 0.023 1/pixel frequency; result without eliminating 0.034 1/pixel frequency, result with the elimination of both frequencies 0.023 and 0.034 1/pixel.

The final analysis tried in this work for the front side of the Shroud is the application of a filtering technique after the elimination of the defects via wavelet filtering.

The following images present the results obtained for the denoised image of the front of the Shroud, after the elimination of the vertical and horizontal frequencies corresponding to defects and after a median filtering process. A comparison is proposed with the same denoised image, but without the use of a median filter. Then, to highlight the differences, some zoomed images are proposed. A final comparison with a Gaussian filtered denoised image is presented.

To see clearly the differences introduced by the median filtering of the Shroud image after the defects elimination, some zooms of the result are proposed. The images obtained after the application of the median filter, on the left, as well as the others on the right, present a contrast and luminance enhancement to highlight results.

Figure 13.1.14: Zoom of the resulting image of the human face after the application of the median filter (left) and without its application (right).

For this detail of the Shroud, the application of a median filter results in an attenuation of the vertical lines crossing the face next to the nose. Some of the high frequency components of the image have been eliminated by means of this filtering process, thus the residual texture becomes less visible. In the red ellipses a vertical line smoothed down in the filtered image on the left is highlighted.

Figure 13.1.15: Zoom of the resulting image of the hands after the application of the median filter (left) and without its application (right).

For this detail of the Shroud, the application of a median filter results in an slightly visible attenuation of the residual texture effect.. In fact the effect of the median filtering is less evident than in the area of the face, as it is visible by comparing the result in figure 13.1.15 with the one in figure 13.1.14. The details underlined with the red curves are meant to highlight some of the attenuated defects in the image on the left, with respect to the image on the right.

Below are presented the effects of a Gaussian filtering.

Figure 13.1.16: Zoom of the resulting details taken from the entire Shroud elaborated of the human face and of the hands after the application of a Gaussian filter with a radius of 1,5.

13.2 Analysis of the back side of the Shroud

In this paragraph the back side of the Shroud is analyzed, following the same method described in the previous section for the front side, that is:

- Dividing the image to be studied into three parts to account for Matlab memory problems;
- Analyzing each of the images found using the prepared software;
- Summing the results obtained for each image 'reattaching' the sub-images and creating the final image.

In this phase of the analysis, the same frequencies found in the sample image taken from the front, have been eliminated. In fact the hypothesis made is that the vertical and horizontal lines have the same frequencies all over the fabric.

A better result could probably be obtained if a sample signal is taken from every sub-image, but in this case it is necessary to remember that some elaboration or arrangement is needed to fix the border lines separating the various sub-images.

Moreover, the burned parts of the Shroud must be excluded from the signal sampled, because otherwise the gray levels do not represent the real horizontal or vertical lines but do just represent a local effect due to fire.

Figure 13.2.1: Subdivision of the back of the Shroud.

Image number	Image Dimensions	Coordinates with respect to the original
1	2301x901	(4199:6499,1200:2100)
2	2000x901	(2199:4198,1200:2100)
3	2198x901	(1:2198,1200:2100)

The following images represent the filtering result of the back side of the Shroud, with the representation of the texture canceled due to the elimination of the frequencies considered to be responsible for the various defects.

Figure 13.2.2: Comparison between the original image impressed on the fabric and the image after the elimination of the frequencies found analyzing the front. On the left, the difference between these two images.

From the images above, one can see that the original image has been smoothed down, but by looking at the zoomed image it also becomes evident that the various vertical and horizontal lines crossing the image impressed have been eliminated or at least they have become less visible.

Figure 13.2.3: Comparison between the original image representing the area next to the head and the final image after elaboration.

Finally, the last operation to complete the study of the back side of the Shroud would be the median filtering. But in this case of the back side analysis, this operation does not seem to be very useful, because there are many blood spots and there are less details of the body image with respect to the frontal part of the Shroud of Turin. The effect of this operation would only be that of smoothing the various spots present, but this is not the purpose of this analysis.

14. Conclusions of the Study

In this work four different images have been analyzed:

1) a sample image taken from the Shroud and representing the human face impressed on it, with the following characteristics:

- a 25 pixel/cm resolution and 661x737 as the number of pixels respectively in each row and column of the image matrix.

2) Another sample image taken from the Shroud, representing a spot, with the following characteristics:

- A 25 cm/pixel resolution and 591x537 as the number of pixels respectively in each row and column of the image matrix.

3) The image of the entire Shroud of Turin, divided into frontal image and back image. Its characteristics are:

- Color image traduced into gray levels with an elaboration done to improve the image quality, by means of Adobe Photoshop CS3 Extended;
- 3498x12090 as the number of pixels respectively in each row or column of the image matrix.

4) The image of the human face without blood traces, with the following characteristics:
- 1495x1281 as the number of pixels respectively in each row and column of the image matrix;
- Color image traduced into gray levels with an elaboration done to improve the image quality and highlight defects and details.

The final results obtained after the elimination of the frequencies corresponding to defects by means of a wavelet filtering and after the application of some other filtering processes to eliminate the residual high frequency components representing the texture are presented in this chapter.

Figure 14.1: Final results after median or Gaussian filtering with a 1,5 radius, respectively on the left and right.

Figure 14.2: Final results after median or Gaussian filtering with a 1,5 radius, respectively on the left and right.

The problem with these images after the Gaussian filtering is that they still show residual effects of the wavelet filtering, especially in the diagonal direction, because there are lines having an inclination of 45° with respect to the horizontal direction. These should be eliminated to enhance the results presented. To show this possibility of further improvement of the filtering technique proposed so far, let us have a look at the following images. They represent:

1) the human face analyzed to eliminate the various texture defects with the wavelet technique and then filtered with a Gaussian filter of radius 3;
2) Its Fourier Transform in the frequency domain.

Figure 14.3: Human face after Gaussian filtering with radius 3 and its FFT.

As explained in chapter four dealing with the FFT analysis of images, the diagonal lines which constitute a cross in the middle of this spectrum represent the diagonal defects rotated of 90°. They represent some frequency components linked to some diagonal fibers of the texture, which should be eliminated to improve the results exposed.

Thus, after wavelet analysis and the application of a filtering technique, also the Fourier approach can be used to eliminate the residual disturbing lines which have an inclination of 45° with respect to the horizontal direction. In this case, the procedure could be similar to the one described in chapter four, but with the use of a specific and dedicated software. The elimination of the extrema of the cross highlighted in the FFT, in figure 14.3 leads to the following result:

Figure 14.4: Extrema of the cross to be eliminated. Below, human face before and after the elimination of some frequencies corresponding to diagonal defects using the FFT.

Figure 14.4 shows that an improvement is possible, using a dedicated software and by means of the FFT, but there still are some difficulties in eliminating the low frequency components of these diagonal defects,

because the frequencies of the image impressed concentrate around the origin of the FFT and could therefore be confused with them.

Considering the results obtained for the human face image without blood traces, it is possible to improve the quality of the image after the filtering process, by applying a technique like the one proposed for the image of the face with blood, presented above. The FFT of this image shows a cross with this shape 'X'; its extrema should be eliminated to cancel the diagonal effects due to the wavelet filtering process. The results obtained after the Gaussian filtering with radius 2, are:

Figure 14.5: Image after wavelet and Gaussian filtering, on the right defects eliminated from the original image by means of the only wavelet filtering.

The results referring to the elimination of the diagonal defects with the highest frequencies (which thus cannot be confused with the frequencies relative to the image impressed) by means of the two-dimensional FFT spectrum are presented below:

Figure 14.6: Image without blood spots after FFT elaboration to eliminate the diagonal effects introduced by wavelet filtering. Above, result after wavelet filtering and the use of the FFT. Below, result after the wavelet and Gaussian filtering, with the final use of the FFT to cancel diagonal defects.

The following image shows the results obtained with the application of the same filtering method applied to the previous sample images, for the entire frontal part of the Shroud.

Figure 14.7: Final results after Gaussian filtering with a 1,5 radius, respectively of the most interesting frontal part of the Shroud.

15. Other Possible Approaches already applied to other studies

The aim of this chapter is to give a brief outline of some analysis approaches used in various scientific fields, which could be applied also to the study of the Shroud. The methods proposed are not only quoted, but also rapidly discussed and compared.

Since the nature of the problem, it is possible to adopt the texture approach to eliminate the lines disturbing the image impressed. Following this idea, a fabric sample is to be considered, then wavelet analysis should be applied over it to identify its frequency components. At this point a research starts among various images, to find those cases in which the frequencies are similar to the frequencies found in the sample.

This is a procedure proposed by Zhang in 2002 and in this case, the implicit hypothesis is that the details in the image have to be clearly visible, therefore the contrast between the background and the details has to be evident.

Unfortunately this is not the case of the Turin Shroud, therefore this technique cannot be applied directly.

Another procedure concerning the astronomical field has been started by Rojo and Harrington in the year 2002. They proceed by coping the original image; in this copy the defect is accentuated by means of a median filter which mediates the gray levels in the vertical direction.

Then the one-dimensional wavelet is applied following rows, to identify the frequency of the defect. Finally, wavelet coefficients are approximated using a Gaussian curve. Using the Inverse Transform, it is possible to obtain an image with highly visible defects. After that, defects can be eliminated by subtracting this image to the original one.

Even if quite interesting, this algorithm also works with the hypothesis of a well defined image, coming out of the background. This is not the case here, therefore the accentuation of the defects causes the loss of some details of the image.

Anyway, it is not possible to determine a unique scale value for all the pixels composing a row of the image, as Pontini found in his study. This effect is a direct consequence of the high variability of the gray levels associated with the vertical and horizontal lines.

The procedure proposed by Harrington and Rojo in 2002 suggests to proceed by subtracting the corresponding pixels of the image before and after elaboration. The wavelet transform can therefore by applied to a detail of the image containing the background.

Then, the gray levels connected to the lines can be extracted, proceeding by subtracting the original image and the image in which the wavelet decomposition level containing the various lines has been eliminated.

After this result, it is necessary to relate the gray levels corresponding to the detail of the image, with the gray levels of any detail of the Shroud.

It is still necessary to solve the problem of how to relate the two. To fix this problem, it is possible to search and determine a correlation between the various lines and their frequency, therefore one has to relate the gray levels due to lines, with the wavelet coefficients of the sample image. The relation between the sample and the part of the Shroud considered is once again determined by means of the wavelet coefficients.

Even if this technique is quite easy to be implemented, the relation between the tones eliminated in the sample image and in the other image needs particular coefficients, to be determined for each different case.

Moreover, it is possible to show that the wavelet transform is not independent on translation, therefore this accounts for a correction which can be adequate just for some of the pixels composing the image.

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